Estimating Time-Varying Network Effects with Application to Portfolio Allocation

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Abstract

In this paper, we characterize a variety of international financial markets as partially correlated networks of stock returns via the implementation of the joint sparse regression estimation techniques of Peng et al. (2009). We explore a number of mean-variance portfolios, with the aim of enhancing out-of-sample portfolio performance by uncovering the hidden network dynamics of optimal portfolio allocation. We find that Markowitz portfolios generally dissuade the inclusion of central stocks in the network, yet the interaction of a stock's *individual* and *systemic* performance is more complex. This motivates us to explore the time-varying correlation of these topological features, which we find are highly market-dependent. Building on the work of Peralta & Zareei (2016), we implement a number of investment strategies aimed at simplifying the portfolio selection process by allocating wealth to a targeted subset of stocks, contingent on the time-varying network dynamics. We find that applying mean-variance allocation to a restricted sample of stocks with daily portfolio re-balancing can statistically significantly enhance out-of-sample portfolio performance in comparison to a market benchmark. We also find evidence that such portfolios are more resilient during periods of major macroeconomic instability, with the results applicable to both developed and emerging markets.

1 Introduction

Almost 70 years have passed since since Markowitz's 1952 paper introduced modern portfolio theory (MPT), otherwise known as mean-variance analysis. Markowtiz provided a mathematical framework for allocating wealth to a portfolio of stocks whilst maximizing expected return for a given level of risk. The covariance matrix of stock returns was shown to be fundamental in determining the optimal portfolio allocation, as portfolio volatility depends on the correlations of pairs of stock returns (Markowitz, 1952). Hence, investors can reduce exposure to idiosyncratic asset risk (volatility) via diversification. However, recent literature has shown that the out-of-sample performance of Markowitz's techniques can be disappointing (DeMiguel et al. 2009).

In the years that have followed, academics have progressed to modelling series of stock returns as a single correlated network, with the bulk of literature implementing such approaches to gain inference on systemic risk within financial markets, particularly after the recent 2008 crisis (Billio et al. 2012), (Diebold & Yılmaz 2014), (Hautsch et al. 2014). However, two recent papers have shown how network-based investment strategies can improve portfolio performance in targeting stocks in certain regions of the network (Pozzi et al. 2013), (Peralta & Zareei 2016). Whilst Pozzi et al. (2013) find that portfolio performance is improved when allocating wealth to stocks located on the periphery of the network, we primarily build on the work of Peralta & Zareei (2016), implementing a variety of investment strategies that take into account the more intricate link between *individual* performance (Sharpe ratio) and *systemic* performance (eigenvector centrality) of the stocks in the network.

We begin our analysis in *Section 2* by representing 4 international stock exchanges as a series of cross-sectional partial correlation networks. We explore the statistics of network theory, with the goal of characterising each stock exchange as a *Graph*, comprised of a set of *Vertices* and *Edges*, via the implementation of the joint sparse regression estimation techniques of Peng et. al (2009). In *Section 3* we introduce mean-variance portfolio theory, uncovering some of the hidden topological features of a series of Markowitz tangency portfolios via modelling them as set of partially correlated networks. This allows us to analyze the complex cross-sectional relationship between a stock's *individual* and *systemic* performance.

In *Section 4* we explore the time-varying correlation between Sharpe ratio and eigenvector centrality (ρ), finding that this relationship between *individual* and *systemic* performance is highly market-dependent and fluctuates over time. When $\rho < 0$, the peripheral stocks in the network (with the least partial correlation) have the highest Sharpe ratios, and so this is where wealth should be allocated for optimal diversification benefits. When $\rho = 0$, there is nothing to infer about the relationship between centrality and individual performance and so peripheral allocation is again preferred to avoid high portfolio variances. However, when $\rho > 0$, the stocks with the highest Sharpe ratios are those that are most central. Hence, a trade-off emerges between high individual performance and systemic risk of the portfolio, and we should invest in the most central stocks only when the correlation is a above a given threshold ($\tilde{\rho}$). We therefore design and implement 3 ρ-dependent investment strategies that simplify the portfolio selection process by investing in a targeted subset of stocks on the network. Although the implementation of a ρ-dependent strategy is first introduced by Peralta & Zareei (2016), we add two additional investment

techniques that utilize optimal mean-variance allocation across the targeted stocks. We implement an out-of-sample investment approach to evaluate the performance of said portfolios. Furthermore, we introduce a benchmark strategy that involves investing the entire unrestricted market, serving as the baseline portfolio to beat. A final *reverse ρ-dependent* strategy is introduced as a control to ascertain whether the results were obtained by chance.

Building considerably on the work of Peralta & Zareei (2016), we carry out these investment strategies on stocks listed on various exchanges from around the world. To ensure varied and robust results, we focus our study in both developed and emerging markets, designing separate portfolios of stocks listed in the UK (LSE), Germany (Deutsche Börse), Brazil (B3) and India (NSE), using daily price data from 01/01/2001 to 31/12/2018 for the 120 most capitalized stocks for each exchange. In *Section 5* we provide a detailed breakdown of our results. Overall, we are interested in determining whether estimating the time-varying topological features of a network can lead to a portfolio simplification process that enhances out-of-sample performance. We conduct further analysis into whether such techniques can yield portfolios that are resilient to periods of major macroeconomic instability, proposing a number of future research questions.

2   Building a Partial Correlation Network

We begin our analysis by representing 4 international exchanges as individual networks of partially correlated stock returns. Network analysis is a powerful tool to represent the interconnections of large multivariate systems, and will form the basis of designing and implementing the various investment strategies that follow. The aim of network analysis is to model the dependence of a series of variables as a single *Graph*, so that we can learn from the network's topological features. A *Graph* (G) is comprised of a set of *Vertices* (ν) connected by *Edges* (ϵ). Graphs can be *weighted*, whereby the thickness of the edge is a function of the level of dependence between two nodes. They can also be *directed*, where an arrow signals a causal relationship from one node to another. In our approach, each stock exchange will be modelled as an unweighted and undirected *Graph* of up to 120 *Vertices* (one for each stock), connected by edges if they display the appropriate level of dependence (Pourahmadi, 2013). Often in the field of Economics and Finance, the measure of dependence depends on the context of the application. We decide to implement *partial correlation* network modelling, providing a linear and contemporaneous network classification that is readily applicable to stock returns and the investment strategies that we use.

$$G = (\nu, \epsilon) \tag{1}$$

2.1 Defining Partial Correlation

The first network definition of partial correlation was introduced by Dempster (1972). Partial correlation measures the cross-sectional linear conditional dependence between two multivariate series y_{it} and y_{jt} controlling for the correlation of other variables in the system. In the context of our work, each y_t represents a series of returns for a particular stock, over a specified time horizon. We choose to compute *partial* correlations between a given pair of stocks, as this accounts for any interference that may be caused by confounding variables (stocks), in terms of noisy correlations with our variables of interest.

$$\rho^{ij} = Corr(y_{it}, y_{jt} | \{y_{kt} : k \neq i, j\}) \tag{2}$$

In network theory, the above equation can be represented in the form provided in *Equation 3*, where an *Edge* between two vertices i and j is formed if there is a non-zero partial correlation between the two series (Pourahmadi, 2013).

$$\epsilon_{PC} = \{\{i, j\} \in \nu \times \nu | \rho^{ij} \neq 0\} \tag{3}$$

Partial correlation is closely related to a linear regression model, where each variable i can be expressed as the linear combination of all of the other variables in the system, as well as an error term u_i. If $\theta_{ij} = 0$, then variables i and j have a zero partial correlation. Furthermore, the partial correlation of y_{it} and y_{jt} is equivalently defined as the linear correlation between the *residuals* of y_{it} and y_{jt} obtained from running separate regression of the two components on all the other variables in the system (Pourahmadi, 2013). In order to build a partial correlation network of returns, we first need to estimate a matrix of partial correlations. Our methodology for estimating this matrix is outlined in Section 2.2.

$$y_{it} = \theta_0 + \Sigma_{i \neq j} \theta_{ij} y_{jt} + u_i \tag{4}$$

2.2 Estimating Partial Correlations Via a Joint Sparse Regression Model

In the bulk of the recent literature that explores network effects, a variety of techniques have been introduced to deal with very large data sets. These techniques can be generally grouped into *subset selection*, *shrinkage estimation* and *dimensionality reduction*, and are all very much applicable to financial data. In the linear regression context (*Equation 4*), subset selection identifies the variables in the system that are the most correlated to the variable of interest. Shrinkage estimation then imposes regularization on the coefficients to create a *sparse* matrix of correlations. The resultant matrix is comprised mainly of zero entries, reducing the variance of the estimates and protecting against spurious

correlations. Dimensionality reduction ensures that the shrinkage estimator is well behaved even when the number of parameters p is much larger than the number of observations T (Hastie, Tibshirani and Wainwright, 2015). In our analysis, we implement the methodology of Peng et al. (2009), who build on *Neighborhood Selection* by developing a smart algorithm to estimate a sparse correlation matrix.

First, it is important to highlight a well known relationship between the partial correlation network and the *concentration matrix*. The concentration matrix K is defined as the inverse of the covariance matrix of the series ($K = \Sigma^{-1}$). It can be shown that the regression parameters θ_{ij} of *Equation 4* can be expressed as a function of the elements of K. This relationship is given in *Equation 5*, where k_{ij} is the *(i,j)* element of the matrix K (Pourahmadi, 2013).

$$\theta^{ij} = -\frac{k^{ij}}{k^{ii}} = \rho^{ij}\sqrt{\frac{k^{ii}}{k^{jj}}} \tag{5}$$

An equivalent formula can be written for the *(i,j)* partial correlation and is provided in *Equation 6*. It shows that the partial correlation network can be entirely characterized by K. If $k_{ij} \neq 0$, then an edge is drawn between the two nodes. Hence, we can reformulate the estimation of the partial correlation network as the estimation of a concentration matrix (Pourahmadi, 2013).

$$\rho^{ij} = -\frac{k^{ij}}{\sqrt{k^{ii}k^{jj}}} \tag{6}$$

As K is assumed to be sparse, we need an estimator that simultaneously selects and estimates the non-zero entries of K. We therefore implement the *Sparse Partial Correlation Estimation* (SPACE) methodology of Peng et al. (2009). They design a smart optimization shooting algorithm that makes use of *LASSO* regression based techniques. This optimization problem is outlined in *Equation 7*, showing that if the diagonal elements of K are known, then we can write an auxiliary linear regression model whose unknown parameters are the partial correlation coefficients. The use of the LASSO technique means that the total number of non-zero parameters is allowed to grow as a function of the number of observations, however the total number must remain relatively small. LASSO is hence being implemented as a *shrinkage* estimator.

The final term in *Equation 7* is known as an *absolute value penalty*. Its role is to shrink some of the estimated coefficients of the auxiliary regression to exact zeros. When $\lambda = 0$ the estimator coincides with least squares, but when λ is sufficiently large, the penalty shrinks certain estimates towards zero, creating a sparse matrix. The λ is often referred to as a tuning parameter, which we set to $0.2 \times T$ after testing over a grid of different values. Furthermore, the ρ^{ij} can be estimated given an estimate for k^{ii}. Given our estimate of ρ^{ij}, we can then re-estimate k^{ii} as the residual variance. The process repeats until the algorithm converges (hence the name *shooting algorithm*). The non-diagonal entries of K *(k^{ij})* can then be estimated from *Equation 6*, and the result is a sparse estimate for K. If $k_{ij} \neq 0$, then an edge is

drawn between the two nodes and the end result is a completed partial correlation network (Peng et al. 2009).

$$\textbf{min} \quad \sum_{i=1}^{n} \left[\sum_{t=1}^{T} \left(y_{it} - \sum_{j \neq i}^{n} \rho^{ij} \sqrt{\frac{\hat{k_{jj}}}{\hat{k_{ii}}}} y_{jt} \right)^2 \right] + \lambda \sum_{i=2}^{n} \sum_{j=1}^{i-1} |\rho^{ij}| \tag{7}$$

Note, that we implement a slight variation of the above in our methodology. We begin by selecting 4 international stock exchanges, and gathering daily stock price data for the 120 most capitalized stocks over the period 01/01/2001 to 31/12/2018. To avoid issues in data collection, we only consider stocks that were active over the full duration of the time series, so that we have 120 stocks for each exchange that were not de-listed, nor did they IPO over the period. We then calculate daily mean returns for the entire portfolio, by summing returns across all of the stocks on a given day ($\sum_{j=1}^{n} \boldsymbol{y_j}$) and dividing by the number of stocks in the portfolio *(n)*. This serves as a proxy for the mean returns of the market in a given day. As a result, we obtain *n+1* vectors; one vector of returns for each of the *n* stocks in the portfolio and a final vector of daily mean returns for the portfolio. For each stock *i*, we regress the vector of returns $\boldsymbol{y_i}$ on the daily means for the portfolio, and for each stock we obtain a vector of residuals $\boldsymbol{u_i}$.

$$\boldsymbol{y_i} = \theta_0 + \theta_1 \frac{\sum_{j=1}^{n} \boldsymbol{y_j}}{n} + \boldsymbol{u_i} \tag{8}$$

The vector of residuals $\boldsymbol{u_i}$ for each stock *i* represents only the exogenous variation in returns for that stock that cannot be explained by the returns of any of the other stocks in the portfolio. This is a simple factor regression that controls for the effect of market. We then run the optimization problem in *Equation 7*, replacing each y_{it} and y_{jt} with u_{it} and u_{jt}. The result is a completed partial correlation network that removes any spurious correlations and is hence better suited to stock price data. In the work that follows, we provide a number of graphical representations of partial correlation networks of stock returns and design various investment strategies based on their time-varying properties.

2.3 Computing Eigenvector Centrality

Once we have built the partial correlation network, defining the importance of a vertex in that network is key when applying our portfolio selection strategies. Typically, we are interested in the level of interconnectedness of a vertex, therefore it is important to measure how *central* the vertex is to the rest of the network. Perhaps the simplest measure of centrality is the *degree* of a vertex, measured by the number of connections that a vertex has to other vertices. However, this classification of centrality tells us nothing about the importance of the other nodes that each vertex is connected to. A more sophisticated definition of centrality is a vertex's *eigenvector centrality* (or *eigencentrality*) score, a concept first introduced by Landau (1885). Eigencentrality scores are built on the notion that if a vertex is connected to a node with many connections, this will contribute more to the score of that vertex than if it were connected to

a node with few connections. Hence, a high eigenvector score means that a vertex is connected to many nodes that are also well-connected. Calculating these centrality scores is pivotal in our implementation of a variety of investment strategies that depend on where certain stocks fit within the topological structure of the network.

As defined by Bonacich (1972), eigenvector centrality assumes that the centrality of a vertex i (v_i) is proportional to the weighted sum of the centralities of its neighbours (ν_j) (*Equation 9*). The Ω is an $n \times n$ *adjacency matrix* with binary inputs that is estimated from the sparse matrix of partial correlations, and is hence also sparse in nature. Quite simply, if the element $\Omega_{ij} = 1$, this represents the existence of a partial correlation between vertices i and j. If $\Omega_{ij} = 0$, then there is an absence of partial correlation between ν_i and ν_j. It is convention to set the elements of the diagonal of Ω to zero to avoid self-loops that convey little meaning; a stock return correlated to itself provides no information in the context of centrality. Note, that this adjustment has no effect on the eigenvectors or eigenvalues (Pourahmadi, 2013). Moreover, the eigenvalue λ can take many values for a non-zero eigenvector solution to exist, however, the Perron-Frobenius theorem shows that the greatest non-zero eigenvalue provides the optimal centrality measure (Perron, 1907), (Frobenius, 1912). In our out-of-sample investment approach (*Section 4*), we estimate a partial correlation matrix and the corresponding centrality scores over a series of 60-day rolling windows, according to *Equation 9*. The result is a vector of eigenvector centralities (one for each vertex) for every day from day 60 until the end of the time series.

$$\nu_i \equiv \lambda^{-1} \Sigma_j \Omega_{ij} \nu_j \tag{9}$$

3 Exploring Modern Portfolio Theory Using Network Analysis

3.1 Mean-Variance Portfolio Theory

In this section we briefly introduce the main results of Markowitz's (1952) mean-variance portfolio theory (MPT). We will rely heavily on the results of this modern portolio theory throughout our work, as in conjunction with network analysis it will help us to uncover some of the hidden dynamics within portfolio allocation and to design various investment strategies accordingly. From now on, we assume that there exists n risky assets, a vector of expected returns (μ), and a covariance matrix of returns (Σ). According to the theory, and in the absence of a risk free asset, *Equation 10* outlines the task of finding a vector of optimal weights w^* that minimizes the variance of the portfolio subject to the given constraint. Allocating wealth according to this strategy is often referred to as the *global minimum variance portfolio* (Back, 2017).

$$\textbf{min} \quad \sigma_p^2 \; = \; w'\Sigma w \quad \textbf{subject to} \quad w'1 = 1 \tag{10}$$

Figure 1: A Graphical Representation of Mean-Variance Theory.
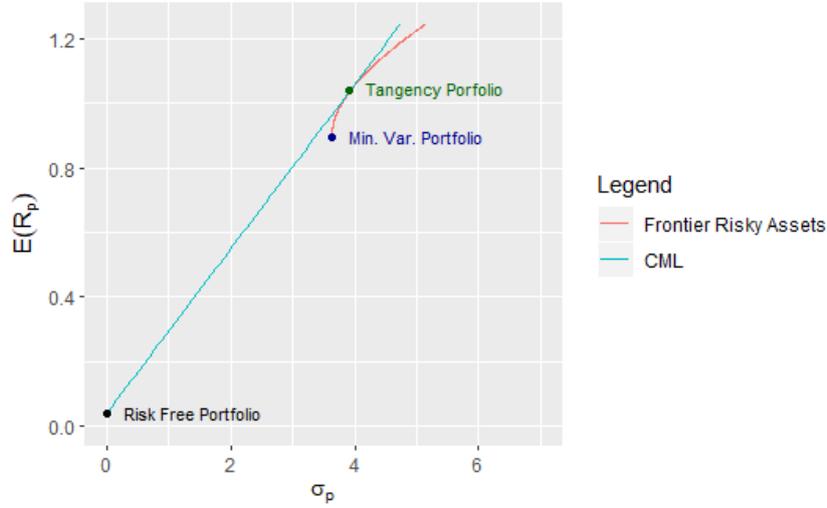
The solution to *Equation 10* is provided in *Equation 11*, where 1 is a vector of 1's and Σ^{-1} is the inverse of the covariance matrix of returns. The resultant vector of optimal weights w^* for the global minimum variance portfolio represents just one out of an infinite set of possible optimal combinations of the n risky assets that lie along the *efficient investment frontier*. The efficient frontier, in the absence of a risk free rate, represents every optimal combination of risky assets that provide the highest possible expected portfolio return for a given level of portfolio risk. We produce *Figure 1* to provide a visual representation of the efficient frontier and the minimum variance portfolio. Hence, according to MPT, all risk-averse investors will invest in a portfolio that lies along this efficient frontier. The w^* in this case simply refers to a combination of the n risky assets that minimizes the volatility of the portfolio. The frontier is therefore truncated, as any point below the minimum variance portfolio is dominated by portfolios that offer a higher expected return for the same level of risk (Back, 2017).

$$w* \;=\; \frac{1}{1'\Sigma^{-1}1}\Sigma^{-1}1 \tag{11}$$

The above theory can be adapted very simply to account for the presence of a risk free asset that exists in the market. This is useful, as it provides us with a much more realistic and applicable model to incorporate into real-world portfolio allocation decisions. Therefore, the portfolio is now comprised of *n+1* assets; n risky assets (with return r_i) and 1 risk free asset (with return r_f). Now, a a *capital market line* (CML) can be extended from the risk free rate tangentially to the efficient frontier, as shown in *Figure 1*. The tangency point is referred to as the *tangency portfolio*. The CML represents an infinite set of optimal portfolios w^* that include combinations of the risk-free asset and the tangency portfolio in various proportions. Any combination of the risk free asset and the tangency portfolio (along the CML) now offers the highest expected return for a defined level of risk. According to MPT, all risk-averse investors will therefore invest in an optimal portfolio along this line. Investing along the CML in a portfolio that lies *below* the tangency portfolio means that some of total wealth is allocated to the

risk free asset. Investing along the CML in a portfolio that lies *above* the tangency portfolio requires shorting the risk free asset and investing more than total wealth in the tangency portfolio. The strategy of investing wholly in the tangency portfolio simply means allocating no wealth to the risk free asset and all wealth to the risky assets. To simplify the portfolio selection process, our mean-variance investment strategies outlined in *Section 4.4* will involve investing all wealth in the tangency portfolio. The solution of optimal weights *w** for investing fully in the tangency portfolio is provided in *Equation 9*. The term *E(R)* represents a vector of expected returns for the risky assets and r_f is the risk free rate in question (Back, 2017).

$$w* = \frac{\Sigma^{-1}(E(R) - r_f 1)}{1'\Sigma^{-1}(E(R) - r_f 1)} \tag{12}$$

3.2   Plotting the Tangency Portfolio as a Partial Correlation Network

In this section we implement the methodology outlined in *Section 2*, to estimate a series of cross-sectional partial correlation networks. We estimate 4 different networks, each comprised of 100 of the most capitalized stocks for 4 different international exchanges, using data spanning the full length of the time horizon (01/01/2001 to 31/12/2018). For each exchange, we then calculate a vector of optimal weights from investing fully in the tangency portfolio, according to *Equation 12*. Our methodology for calculating daily stock returns and risk free rates will be addressed in detail in *Section 4.3*. Note that in finding *w**, we also use data spanning the entire time series. We then graph each partial correlation network (*Figure 2*), weighted by the optimal weights of the tangency portfolio. To do so, we make use of R's *igraph* package, where the darker the shade of the vertex represents a higher weight allocated to that stock.

The work of Peralta & Zareei (2016) and (Pozzi et al. 2013) shows that stocks that receive a large amount of wealth in an optimal Markowitz portfolio tend to be placed in the periphery of the network. We corroborate these findings with our graphs plotted in *Figure 2*, although we find that the result is market-dependent. For the majority of international exchanges analysed, the amount of wealth optimally allocated to less central stocks in the network is much higher when compared to allocation to more central stocks. This is shown by the majority of darker nodes being placed towards the outskirts of the network, meaning that they receive a higher optimal weight in the tangency portfolio. The intuition behind this result is that very central stocks lead the general market movement and hence reduce the portfolio's *systemic performance* by increasing its variance.

The above result is clearer for certain countries than for others, and so may be described as market-dependent. The strategy of optimally allocating more weight to less central stocks is more evident for the UK, Brazil and India, and less so for Germany. However, in general, graphing the partial correlation network shows that investing according to MPT dissuades the inclusion of highly central stocks in an

Figure 2: The Tangency Portfolio as a Partial Correlation Network.
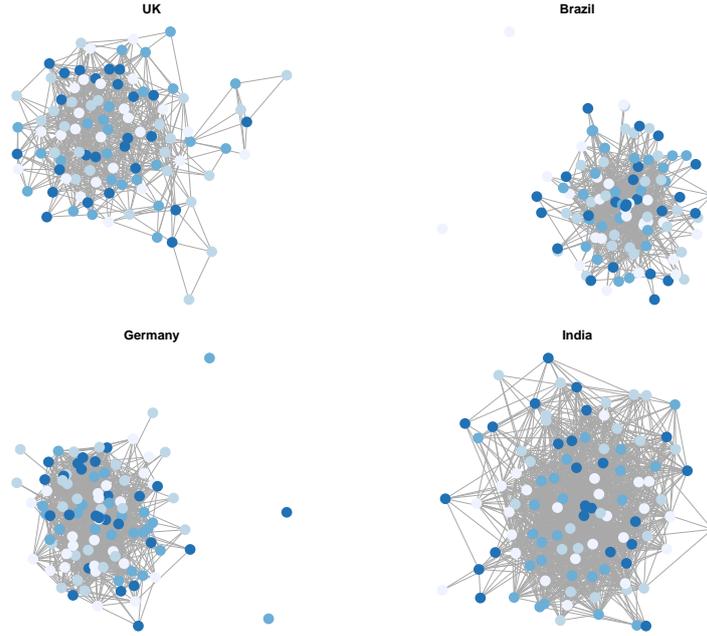

optimally designed portfolio. Note that investing in lowly central stocks can be defined as investing in stocks with poor *systemic performance.*

Furthermore, the partial correlation network allows us to analyse the hidden dynamic between *systemic* performance and *individual* performance (Sharpe Ratio) for a given portfolio. This is outlined in *Figure 3*, where the darkness of the node increases in the weight allocated to that stock under the optimal allocation of the tangency portfolio. For stocks listed on the UK's London Stock Exchange, it is clear that a higher weight is allocated to stocks with lower eigencentrality scores and higher Sharpe ratios. This is clear from the cluster of darker nodes in the upper-left corner of the graph. Stocks with low Sharpe ratios are in fact hardly given any weight and so, for this market, Markowitz allocation seems to prioritize *individual* performance over *systemic* performance. This is clearly different from the NSE of India, where investing more in stocks with low centrality is prioritized over individual performance. In Brazil, there are a larger number of stocks that have good individual and low systemic performance. Weight seems to be spread across the stocks fairly evenly, possibly because highly central stocks are very few in number. For Germany, tangency portfolio allocation leads to a fairly even distribution of wealth across the 100 stocks.

What we learn from *Figure 3* is that the hidden dynamics of portfolio allocation can be complex and seem to be market-dependent. Although typically under mean-variance allocation wealth is allocated to the least central stocks, this is not always the case. It is important to note that so far we have provided only a cross-sectional view of the partial correlation network in *Figure 2* and *Figure 3*. The mixed results we obtain are likely to be explained by analysing the time-varying characteristics of the network. In reality, there will be periods of time in which the most central stocks will also have the highest individual performance. This may lead to a trade-off in portfolio allocation, where more weight is given to better performing central stocks despite the risk of high portfolio variances. Understanding this time-varying

Figure 3: Optimal Weights for Tangency Portfolio Strategy.
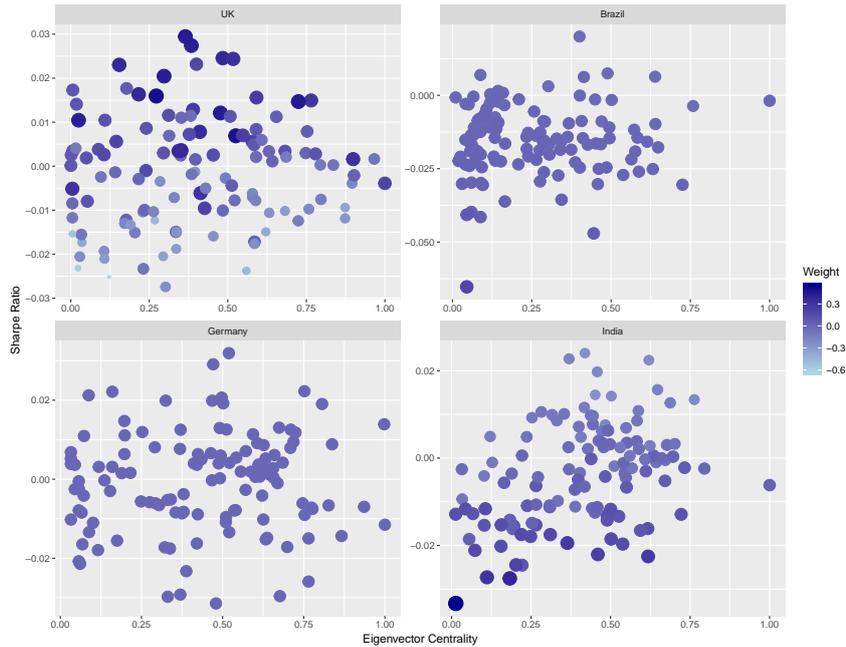
correlation of *individual* and *systemic* performance is the motivation for the investment strategies that follow. We look to simplify the portfolio allocation process by investing in only a targeted group of stocks, contingent on the dynamic topological features of the network. Our objective is to achieve higher out-of-sample portfolio performance by a method of intelligent stock-selection, hence benefiting more from these time-varying correlations than traditional investment methods.

4 Implementing a ρ-dependent Investment Strategy

4.1 Defining ρ

In keeping with the work of Peralta & Zareei (2016), we analyse the time-varying correlation (ρ) between the *individual* performance (Sharpe ratio) and the *systemic* performance (centrality) of each stock, and how this affects optimal portfolio choices. When $\rho < 0$, the peripheral stocks in the network (with the smallest partial correlations) have the highest Sharpe ratios, and so this is where wealth should be allocated for optimal portfolio diversification benefits. In restricting investment to a subset of peripheral stocks, high individual stock performance is achieved with low portfolio risk. Furthermore, when $\rho = 0$, there is nothing to infer about the relationship between centrality and individual performance, and so peripheral allocation is again preferred to avoid high portfolio volatility. However, when $\rho > 0$, the stocks with the highest Sharpe ratios are those that are the most central. Hence, a trade-off emerges between investing in stocks with a high individual performance and increasing the systemic risk of the portfolio. Investing in central stocks with high individual Sharpe ratios reduces systemic performance if returns are highly correlated with the returns of other central stocks. Therefore, we should invest in the most central stocks only when the correlation ρ is a above a pre-determined threshold ($\tilde{\rho}$).

4.2   Methodology

To implement our ρ-dependent investment strategy, we use an out-of-sample approach, whereby the eigenvector centrality and Sharpe Ratio are calculated for each stock over a series of 60-day rolling windows. Each day, the 60-day window shifts by one day into the future, until the end of the time series is reached. Hence, an eigenvector centrality score and Sharpe ratio is calculated for every stock, each day, from day 60 to the end of the time series. The daily individual eigenvector centralities are calculated using the methodology described in *Section 2*. We estimate a partial correlation network using data from the past 60 days and compute eigencentrality scores according to *Equation 9*. To calculate individual Sharpe ratios, we calculate the 60-day mean of daily returns for each stock, subtract the corresponding 60-day mean of daily "risk-free" rates, and divide by the 60-day mean of the "daily" standard deviation of returns. Note that we proxy for daily standard deviation by taking the standard deviation of returns over the previous 60 day window and dividing by $\sqrt{60}$.

$$Sharpe\ Ratio_i = \frac{\bar{R}_i - \bar{R}_f}{\tilde{\sigma}_i} \tag{13}$$

This methodology is applied to each of the 4 exchanges in our data set, each containing a time series of 120 stocks spanning from 01/01/2001 to 31/12/2018. Note, that to proxy for the "risk free" rate, we download daily data from Thompson Reuters of 3-month Treasury bill yields for each of the countries in which the exchanges are domiciled. We convert each of the 3-month rates to daily values, so that we have a complete time-series of daily risk free rates for 4 different countries from 01/01/2001 to 31/12/2018. We use the 3-month yield as the risk free proxy for a number of reasons. Firstly, in many of the countries under evaluation, 3-month Treasury bills carry almost no default risk as they are fully guaranteed by the government or central bank in question. Secondly, when deciding on the maturity, it is important to consider the time period of investment. Although we conduct our investment strategies over a 19-year period, we update the parameters of our model *daily* and change our investment strategy accordingly. Although the investment horizon is rather long, a continuously evolving strategy that is built on the past 60 days of data indeed calls for the use of a short-term risk free rate.

From the methodology described above, for each exchange we obtain a vectors of 120 individual eigenvector centralities and Sharpe ratios per day. We then simply compute the correlation between these two vectors to obtain a daily value of ρ for each of the exchanges. This value of ρ represents the correlation between all of the Sharpe ratios and centralities in each of the portfolios for any 60-day period. Hence, we obtain a value of ρ from day 60 until the final date in the time series (31/12/2018). This will allow us to implement a ρ-dependent investment strategy, whereby the composition of each portfolio is adjusted daily.

4.3 Visualizing the Time-Varying ρ

Figure 4: Time-Varying Correlation of Sharpe Ratio and Centrality (ρ).
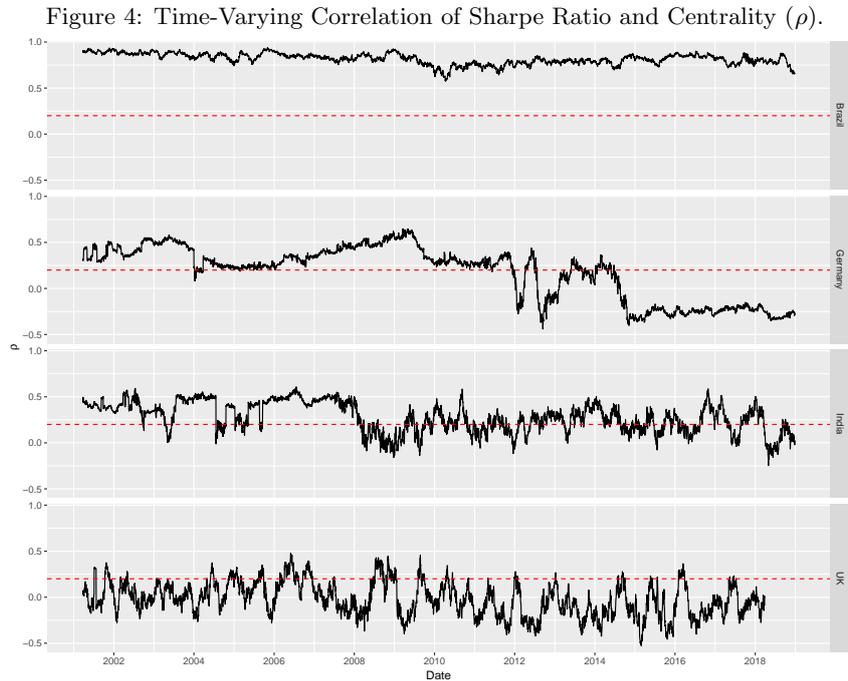
In *Figure 4* we plot a time-series of ρ, showing the correlation between *systemic* and *individual* performance for 120 stocks for each of the 4 exchanges. Firstly, it is clear that ρ is time-dependent and can fluctuate between positive and negative values, and take values above and below a pre-defined threshold of 0.2. This means that there will clearly be occasions where we invest in the least central stocks ($\rho \leq 0.2$) and occasions where we invest in the most central ($\rho > 0.2$). The exact methodology for this process will be explained in the following *Subsection 4.4*. Note that in our analysis we assume a threshold of $\tilde{\rho}$ = 0.2, in keeping with the work of Peralta & Zareei (2016). In their study, they randomly generate 120 data sets of 150 stocks each with values of ρ ranging from -0.20 to 0.45. They conclude that $\tilde{\rho} = 0.2$ generates the highest out-of-sample Sharpe ratios, and is high enough to warrant targeting stocks in the network with the highest centrality scores, despite risking higher portfolio variances.

Moreover, it is also evident from *Figure 4* that there are distinct differences across markets with respect to the evolution of ρ. The value of ρ for Brazil never drops much below 0.75, whereas the ρ of the UK never reaches 0.5. Again, there are clear differences in the path of ρ for Germany and India, and no two portfolios seem to follow the same pattern. This is made further evident from *Table 1*, showing the percentage of days for which the value of ρ falls below the given threshold of 0.2. For the portfolio of Brazilian stocks, the tendency is for the ρ to be much higher over the entire time horizon, with the ρ never falling below the threshold. Hence, within the Brazilian portfolio we will only ever invest in the most central stocks on any given day. The opposite is true for the UK, where for 87% of days we will be investing in the least central stocks. For Germany (India), we will invest in the least central stocks on 36% (34%) of trading days.

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Table 1: Percentage of Days $\rho \leq 0.2$

Country	% Days
UK	87.32
Brazil	0.00
Germany	35.60
India	33.66

4.4 Out-of-Sample Approach

We implement our out-of-sample approach as follows. For each of our markets, we have a portfolio of 120 stocks. Starting from day 60 until 31/12/2018, we calculate a daily ρ for each of the portfolios using a 60-day rolling window approach described in *Section 4.2*. For each market, we then rank the stocks according to their eigenvector centrality score. When the daily $\rho \leq \tilde{\rho}$, we allocate wealth only to the 20 *least* central stocks in the network according to that centrality score. If $\rho > \tilde{\rho}$, we allocate wealth only to the 20 *most* central stocks in terms of their eigenvector centrality. This approach can be described as "out-of-sample", as we are calculating values of ρ and eigencentralities at the close of day t (based on the previous 60 days) and investing in a restricted subset of 20 stocks at the open of day $t+1$. We then hold these 20 stocks over the course of the trading day, evaluating portfolio performance according to the returns of those stocks over day $t+1$. At the end of the day $t+1$, we repeat the process again. We calculate the value of ρ for that day and the eigencentralities, before applying the same strategy again for a (potentially) new set of 20 stocks at the open of day $t+2$. We evaluate portfolio performance according to stocks returns over the duration of the trading day $t+2$, and this process is repeated until the final date in the time series is reached. The decision to restrict the *ρ-dependent* portfolios to 20 stocks each day is based on the findings of Desmoulins-Lebeault and Kharoubi-Rakotomala (2012), who show that most of the benefit of diversification stems from investing in just 20 assets.

The first investment strategy that we use when investing in the 20 targeted stocks is the *Naïve* wealth allocation strategy introduced by Peralta & Zareei (2016). When $\rho \leq \tilde{\rho}$, we allocate wealth evenly across the 20 least central stocks according to a 1/N rule, where N represents the number of stocks in the restricted portfolio (20). Conversely, when $\rho > \tilde{\rho}$ we spread wealth evenly accross the 20 most central stocks. We see the value in adopting this strategy, as it allows us to evaluate portfolio performance according to what turns out to be a very simplistic diversification procedure. Furthermore, it has been shown that a 1/N allocation process can even lead to superior out-of-sample performance than the more complex method of Markowitz diversification (DeMiguel et. al, 2009). However, unlike Peralta & Zareei (2016), we decide to also implement investment strategies according to Markowitz's mean-variance theory. We consider an investment strategy whereby instead of using the *Naïve* 1/N diversification rule, we invest in the tangency (market) portfolio for the stocks that we are considering. This mirrors a more sophisticated investment approach that is more likely to be implemented by portfolio managers in the real world. We are therefore keen to explore whether using MPT in conjunction with a portfolio-simplification strategy can achieve superior out-of-sample performance compared to *Naïve* investment methods. As explained in *Section 3.1*, the solution to finding the optimal weights w^* for allocating wealth wholly to the tangency portfolio in the presence of a risk free asset is provided in *Equation 9*.

In our MPT investment approach, we follow a similar initial out-of-sample procedure to that of our *Naïve* investment strategy. We first calculate a daily value for the ρ of the portfolio of 120 stocks, using 60-day rolling windows. We therefore again obtain a value of ρ every day from day 60 until the 31/12/2018. We then select the 20 targeted stocks based on the exact same criteria as previously. When $\rho \leq \tilde{\rho}$, we invest in the 20 least central (ranked by eigenvector centrality) and we invest in only the 20 most central when When $\rho > \tilde{\rho}$. The only adaptation is that once the 20 stocks are targeted at the close of day t, we now compute a matrix of the covariance of returns for those stocks over the previous 60 days and calculate the inverse. We also compute a vector of excess returns for day t by subtracting the daily risk free rate from the daily return of each stock. We can then calculate the optimal weights of the tangency portfolio w* for day t, according to *Equation 9*. We hence allocate wealth to those 20 stocks at the open of day $t+1$, weighted by w* from day t. At the close of day $t+1$, we select the next 20 stocks based on the new ρ, recalculate w* based on the previous 60 days, and invest in these 20 stocks at the open of day $t+2$ based on the new optimal weights. We implement this approach separately across across all of the markets in our data set, each day recording the returns of our portfolios. Note that we refer to this investment strategy as *Tangency* in the work that follows. We also implement a very similar investment strategy, which we refer to as *Tang. Lim*. We follow the exact same procedure as outlined in our *Tangency* approach with one difference; we add a short-sale constraint to the optimization problem. In particular, we limit our investment strategy to allocating no more than 50% of total wealth to shorting a given stock at any point in time. This is to employ a more realistic mean-variance investment procedure that is designed to avoid potentially unlimited losses and to hence keep portfolio variances stable.

We also introduce one final investment strategy, which we refer to in our analysis as *Market*. This approach involves simply investing in all 120 stocks for each of the international exchanges irrespective of the time-varying ρ. Thus, this strategy can be referred to as *non ρ-dependent*, as we apply the 1/N rule to all 120 stocks and hold them in equal weight for the entirety of the investment horizon. As this strategy does not involve stock selection, it can therefore be taken as a proxy for investing in the entire market. This strategy hence serves as the benchmark to beat in our out-of-sample implementation of our 3 ρ-dependent stock selection strategies. The intuition behind the inclusion of the *Market* strategy is to ascertain whether estimating the time-varying nature of network effects leads to a portfolio simplification process that enhances out-of-sample performance with respect to the market. Finally, we introduce a *reverse ρ-dependent* control strategy that enables us to determine whether our results are obtained chance. The strategy simply reverses the criteria for investing in the targeted group of stocks, so that when $\rho \leq \tilde{\rho}$ we invest in the *most* central stocks, and we invest in the *least* central stocks on occasions where $\rho > \tilde{\rho}$.

5   Results

In this section, we apply our 4 investment strategies (*Tangency, Tang. Lim, Naive, Market*) to each of the 4 international exchanges and evaluate their out-of-sample performance. We also implement the reverse-ρ procedure on each of the 3 ρ-dependent strategies, serving as a control. We compute the 12-month rolling Sharpe ratio for each of the portfolios by considering a series of 252-day rolling windows with steps equal to 1 day in length. To do so, each day we take the 252-day mean Sharpe ratio for the portfolio based on daily portfolio Sharpe ratios that are calculated previously in *Section 4.4*. Note that

computing annualized Sharpe ratios is common practice in the evaluation of portfolio performance.

In the subsections that follow, we plot the 12-month rolling Sharpe ratios to provide a detailed country-by-country analysis of portfolio performance. First however, we provide some descriptive statistics of our results (*Table 2* and *Table 4*) showing the mean 12-month rolling Sharpe ratio broken down by country and strategy. With the exception of Brazil, all 3 ρ-dependent strategies and the *Market* benchmark deliver positive Sharpe ratios, with the vast majority statistically significantly different from zero. For stocks listed in the UK, Germany, and India, the *Naïve* portfolio fails to deliver enhanced out-of-sample performance, as these portfolios performs statistically significantly worse than the benchmark. This implies that the simple strategy of investing in the market in equal weight is optimal when compared to the more complicated strategy of a daily portfolio re-balancing and 1/N allocation to a target group of stocks contingent on the time-varying ρ. This finding is at odds with the results of Peralta & Zareei (2016), who show that a *Naïve* ρ-dependent strategy does indeed beat the market. Our differing findings may be due to a number of factors; primarily that we consider a much greater investment time horizon, focused on different markets, and implemented more modern techniques in our sparse estimation of partial correlations.

Focusing on our *Tangency* strategy for the UK, Germany, and India, *Table 2* shows that performance is no longer strictly dominated by the benchmark. In fact, for the UK and Germany, the performance of these portfolios is statistically significantly greater than that of the simplistic strategy of investing in the entire market. This shows that a ρ-dependent strategy that simplifies portfolio selection can indeed lead to superior out-of-sample returns when investing in the tangency porfolio. This result is particularly salient for Germany, where the mean Sharpe ratio of 13.8 is not only statistically significantly different from zero, but is also almost 63 times greater that of the benchmark. However, this is not the case for India, where the mean Sharpe ratio does not surpass the benchmark and is not significantly different from zero. Furthermore, the results of the *Tangency* allocation via the reverse-ρ control method in *Table 4* show that our results are not obtained by chance. When reversing the investment criteria, portfolios for the UK, Germany, and India all record negative Sharpe ratios that are significantly different from zero, performing far worse than the market benchmark. This provides an additional level of robustness to the previous findings.

A similar pattern emerges when analysing the performance of the *Tang. Lim* investment strategy. In this case, the portfolios for India and Germany statistically significantly outperform the benchmark, and the positive Sharpe ratios are different from zero also with statistical significance. Note that in this instance, the German *Tang. Lim* portfolio outperforms the *Market* benchmark by more than 100 times. *Table 4* shows that the reverse-ρ *Tang. Lim* portfolios for the Germany and India yield negative Sharpe ratios, significantly under-performing the market benchmark. This highlights that the results of the ρ-dependent *Tang. Lim* strategy are not obtained by chance. However, it is important to note that the results of the *Tang. Lim* strategy for the UK are fairly disappointing. Although the mean Sharpe ratio for the portfolio is positive, it is shown to be non-significantly different from zero, performing significantly worse than the benchmark. Rather surprisingly for the UK, the reverse-ρ *Tang. Lim* strategy shown in *Table 4* delivers a positive Sharpe ratio that actually significantly enhances performance according to the benchmark This is an unexpected finding that will be explored in due course.

Despite the results being mixed for the UK, Germany, and India, we learn that the following a Markowitz ρ-dependent strategy not only simplifies the portfolio allocation process but it can lead to significantly superior out-of-sample performance than simply investing in the market via a 1/N diversification rule, and the results are robust in the majority of cases. These findings are at odds with those of DeMiguel et al. (2009) who show that the opposite is true. However, for the stocks listed in Brazil, the results are far less clear. As every strategy returns a negative Sharpe ratio, the performance of the portfolios cannot be compared to one another as interpreting negative Sharpe ratios conveys little meaning. Furthermore, the negative Sharpe ratios for the Brazilian *Tangency*, *Naïve*, and *Market* portfolios are all significantly less than zero. As the *Tang. Lim* strategy returns a Sharpe ratio that is not significantly different from zero, this is taken to be the optimal strategy that enhances portfolio performance compared to all others. This result is further supported in *Table 4*, as the Sharpe ratio for the *reverse-ρ Tang. Lim* strategy is statistically significantly less than zero.

Table 2: Mean 12-month Rolling Sharpe Ratios.

Country	Tangency	Tang. Lim	Naive	Market
UK	0.2416***	0.0206	0.0340**	0.0780***
	(0.0153)	(0.0151)	(0.0151)	(0.0151)
Brazil	-0.0474***	-0.0072	-0.2498***	-0.6503***
	(0.0151)	(0.0151)	(0.0153)	(0.0166)
Germany	13.8069***	22.4643***	0.0643***	0.2204***
	(0.1482)	(0.2403)	(0.0151)	(0.0152)
India	0.0185	0.1635***	0.0469***	0.0970***
	(0.0151)	(0.0152)	(0.0151)	(0.0151)

*p<0.10,**p<0.05,***p<0.001 $H_0 : SR = 0$

Table 3: Mean 12-month Rolling Sharpe Ratio 2006-2009.

Country	Tangency	Tang. Lim	Naive	Market
UK	0.2711***	0.0693**	-0.4008***	-0.3622***
	(0.0370)	(0.03641)	(0.0378)	(0.0375)
Brazil	-0.5401***	-0.3060***	-0.0184	0.2410***
	(0.0389)	(0.0372)	(0.0364	(0.0369)
Germany	0.2935***	0.2824***	-0.6197***	-0.4863***
	(0.0371)	(0.0371)	(0.0397)	(0.0385)
India	0.3620***	0.4355***	-0.3082***	-0.2962***
	(0.0375)	(0.0380)	(0.0372)	(0.0371)

*p<0.10,**p<0.05,***p<0.001 to $H_0 : SR = 0$

Table 4: Mean 12-month Rolling Sharpe Ratios (reverse-ρ).

Country	Tangency	Tang. Lim	Naive	Market
UK	-0.7074***	0.2502***	0.1536***	0.0780***
	(0.0171)	(0.0155)	(0.0154)	(0.0153)
Brazil	-203.8342***	-1.1110***	-119.4047***	-0.6503***
	(2.19)	(0.0194	(1.2833)	(0.0168)
Germany	-107,6349***	-0.2485***	0.04386***	0.2204***
	(1.1660)	(0.0155)	(0.0153)	(0.0154)
India	-61.5554***	-0.4624***	0.1473***	0.0970***
	(0.6580)	(0.0160)	(0.0153)	(0.0153)

*p<0.10,**p<0.05,***p<0.001 to $H_0 : SR = 0$

Table 5: Mean 12-month Rolling Sharpe Ratio 2006-2009 (reverse-ρ).

	Tangency	Tang. Lim	Naive	Market
UK	0.5954***	0.6074***	-0.3590***	-0.3622***
	(0.0395)	(0.0395)	(0.0375)	(0.0375)
Brazil	-280.1428***	-0.3374***	-170.9398***	0.2410***
	(7.2000)	(0.0420)	(4.3960)	(0.0369)
Germany	-298.8850***	-0.8142***	-0.6368***	-0.4863***
	(7.6866)	(0.0420)	(0.0399)	(0.0385)
India	-60.3890***	-0.5256***	-0.2215***	-0.2962***
	(1.5435)	(0.0388)	(0.0368)	(0.0372)

*p<0.10,**p<0.05,***p<0.001 to $H_0 : SR = 0$

5.1   UK

In *Figure 5*, we plot the 12-month rolling Sharpe Ratio for each of the investment strategies for the UK stocks. It should be noted that the *Naïve* strategy and the *Market* benchmark possess a strong positive correlation, with the trajectory of the Sharpe ratios following almost exactly the same path. This is an interesting result that suggests that a simplistic ρ-dependent 1/N diversification strategy has little effect in terms of enhancing out-of-sample portfolio performance. This is in line with the findings in *Table 2* that suggest that the ρ-dependent 1/N approach yields statistically significantly inferior portfolio performance when compared to the benchmark. Similarly, the two Markowitz allocation strategies (*Tangency* and *Tang. Lim*) also have a portfolio performance that is positively correlated to one another, yet this correlation is weaker than in the previous case.

An important result for the UK is the apparent resilience of the two mean-variance strategies during periods of major global financial instability. During the Financial Crisis of 2008-2009, the Sharpe ratio of these two portfolios is almost always positive. This is a great deal different to the results of the 1/N investment strategies, where Sharpe ratios are always negative over the same period. This is shown clearly in *Figure 6*, where we calculate 12-month rolling Sharpe ratios for the period 2006-2009 to analyse

the effect of each strategy during the Financial Crisis. The Sharpe ratios for the mean-variance portfolios are positive for a much longer period than the 1/N strategies. This suggests that these portfolio selection strategies may provide a buffer during major financial turmoil. This is further evident from the results in *Table 3*, where we compute mean 12-month rolling Sharpe ratios from the years 2006-2009. Whilst the *Naïve* and *Market* Sharpe ratios are negative during this period and statistically significantly different from zero, both Markowitz ρ-dependent portfolios demonstrate a positive performance, with both the *Tangency* and *Tang. Lim* portfolios improving on the benchmark portfolio with statistical significance. Surprisingly however, *Table 5* shows that the *reverse-ρ* Markowitz strategies significantly outperform both of the 1/N approaches during the period 2006-2009. Furthermore, these *reverse-ρ* mean-variance strategies yield a higher mean Sharpe ratio than any other of the UK portfolios. Although the *reverse-ρ* control strategy performing optimally is surprising, it may be rationalized if the portfolios were comprised of a number of successful short positions in the most central stocks that were most likely to be adversely affected during the Financial Crisis. Although this is purely a hypothesis, it presents an area for future research.

Moving along in the time series, *Figure 5* shows that during the European Soverign Debt Crisis that began in January 2010, the *Tangency* strategy is the only portfolio to show a clear upward trajectory in Sharpe ratio. The *Tang. Lim* indeed shows a downward trajectory, but not as steep of a decline as the two 1/N strategies. Furthermore, after the UK's decision to leave the EU in June 2016, the UK has suffered significant economic instability that is still on-going. The two mean-variance portfolios both demonstrate a steep rise in Sharpe ratios that has remained positive ever since. This is in stark contrast to the *Naïve* and *Market* portfolios where a slighter initial increase in Sharpe ratio has been followed by a speedy decline to values below -1.0 at 31/12/2018. This is further evidence of a resilience of the ρ-dependent mean-variance strategies to macroeconomic instability.

5.2   Germany

In *Figure 7* we plot the 12-month rolling Sharpe Ratio for each of the investment strategies for the German stocks. As we found for the UK, the *Naïve* strategy and the *Market* benchmark for Germany also possess a strong positive correlation, with the trajectory of the Sharpe ratios following an almost identical path. This again suggests that a simplistic ρ-dependent 1/N diversification strategy has little effect in terms of enhancing out-of-sample portfolio performance. This is also in line with the findings in *Table 2*, implying that implementing a ρ-dependent 1/N procedure produces statistically significantly inferior portfolio performance when compared to the benchmark. Similarly to that of the UK, the two mean-variance strategies also show a portfolio performance that is positively correlated, but this correlation is made weaker than that of the 1/N portfolios due to an extremely steep negative decline in Sharpe ratio for the *Tangency* strategy at the begining of 2004. This results in a rolling Sharpe ratio that falls below -100, however this is avoided with the inclusion of short-sale constraints in the *Tang. Lim* portfolio. Avoiding these types of major portfolio variances was the main motivation for the introduction of such limits, and shows the benefit of implementing additional constraints to the targeted stocks.

Moreover, the Markowitz ρ-dependent portfolios are extremely resilient to any major macroeco-

nomic upset. This is shown in *Figure 8*, where again compute 12-month rolling Sharpe ratios for the period 2006-2009 to analyse the effect of the Financial Crisis on the performance of each portfolio. We see that for allocating wealth according to the *Naïve* strategy and investing in the *Market* both yield poor results over the period 2006-2009. For these portfolios, Sharpe ratios are consistently negative and fall below -2.0 in 2007. This is in contrast to the targeted mean-variance portfolios, where Sharpe ratios become negative for a much shorter period from mid-2008 to mid-2009. These portfolios seem to recover much quicker, with the Sharpe ratios becoming positive in the final quarter of 2008 and remaining relatively high until 2010. This is made further evident from the results in *Table 3*, showing the mean 12-month rolling Sharpe ratios for each portfolio for the period 2006-2009. The approach of targeting stocks with the mean-variance strategies leads to statistically significantly superior portfolio performance than the *Naïve* allocation, and provide a means of significantly beating the market during this period of extreme hardship for the German economy. This result is strongly supported by the poor performance of the *reverse-ρ* mean-variance strategies over the same period, displayed in *Table 5*. When reversing the investment criteria, both German mean-variance portfolios return negative Sharpe ratios that are statistically significantly different from zero.

5.3   Brazil

In *Figure 9* we plot the 12-month rolling Sharpe Ratio for each of the investment strategies for the Brazilian stocks. In contrast to the developed economies of the UK and Germany, the *Tangency* and *Tang. Lim* strategies share a strong positive correlation over the time period, where as the 1/N allocation strategies show a positive but weaker correlation. Furthermore, the results of *Table 2* show that in fact all portfolios give negative mean Sharpe ratios, including the strategy of investing in the entire market. However, as the *Tang. Lim* strategy is the only portfolio with a Sharpe ratio that is not significantly different from zero, this portfolio can be said to enhance out-of-sample portfolio performance over the entire time horizon. This result is robust in the sense that the reverse-ρ *Tang. Lim* portfolio yields a negative Sharpe ratio that is significantly less than zero. Furthermore, it is clear that post-2013, implementing Markowitz allocation on a targeted set of stocks does lead to a higher portfolio performance in comparison to the *Naïve* strategy and investing in the *Market*. Moreover, the ρ-dependent mean-variance strategies seem to shield the portfolios from bearing negative Sharpe ratios during Brazil's severe economic crisis of 2014-2016. This further highlights the potential benefit of implementing such portfolio simplification strategies during periods of major economic distress, and shows that such strategies may also be applicable to emerging markets.

In *Figure 10* we focus our analysis on the period 2006-2009 to evaluate the performance of the portfolios during the global Financial Crisis. We see that the performance of all of the portfolios is similar over the period, however the results of *Table 3* show that the Brazilian market was largely unaffected by the global crisis, as the investing in the entire *Market* delivers a positive mean Sharpe ratio that is statistically significantly different from zero. In fact, all other strategies provide a poor comparative performance, implied by the negative mean Sharpe ratios, significantly different from zero for the two mean-variance portfolios. As the Brazilian economy was fairly unaffected during this turbulent period, the ability of the ρ-dependent Markowitz portfolios to enhance relative out-of-sample performance may be limited to instances of severe economic downturns, particularly for the Brazilian market.

5.4 India

In *Figure 11* we plot the 12-month rolling Sharpe Ratio for each of the investment strategies for the Indian stocks. It is evident that the two 1/N allocation strategies share a strong positive correlation, and so a ρ-dependent strategy that allocates naïvely to a targeted group of stocks does not seem to enhance portfolio performance compared to the market benchmark. The two Markowitz strategies also bear a fairly strong positive correlation to one another, however this relationship weakens from 2015 onward. The results in *Table 2* show that out-of-sample performance of the *Tang. Lim* portfolio is statistically significantly greater than that of the benchmark, however the *Tangency* strategy without short-sale limits performs significantly worse in terms of mean Sharpe ratio. Analyzing the results of the *reverse-ρ* approach in *Table 4*, we see that both Markowitz portfolios deliver negative Sharpe ratios that are significantly less than zero. This supports the ability of the *Tang. Lim* strategy to enhance out-of-sample performance compared to the benchmark, and we can be confident that both ρ-dependent mean-variance strategies lead to positive Sharpe ratios.

Unlike the previous 3 markets analyzed, the Indian economy suffered no major financial crisis nor any recessionary period during the time horizon under consideration. In fact, annual GDP growth for India only falls marginally below 4% on two occasions from 2001 to the end of 2018. The first instance of this is in 2002, where the mean-variance strategies outperform the 1/N allocation strategies, demonstrated by their positive Sharpe ratios. This shows that targeting stocks via a ρ-dependent approach in conjunction with Markowitz allocation may lead to more resilient portfolio performance. This is further highlighted in *Figure 12*, where we plot 12-month rolling Sharpe ratios for the 4 portfolios from 2006-2009 to analyze the impact of the 2008 Financial Crisis, a period when Indian GDP growth is more than halved. We see that the two Markowitz portfolios deliver mostly positive Sharpe ratios over the period. This is in juxtaposition to the *Naïve* portfolio and the market benchmark, which both realize negative Sharpe ratios from mid-2008 to mid-2009. This is further highlighted in *Table 3*, showing that the *Tangency* and *Tang. Lim* portfolios provide a statistically significantly superior performance compared to the 1/N strategies. These results are supported by findings of the *reverse-ρ* control strategy shown in *Table 5*. The two mean-variance strategies now deliver negative Sharpe ratios that are significantly less than zero, showing that our results are not coincidental. This further supports the notion that our method of stock targeting with Markowitz allocation can create portfolios that are more robust to macroeconomic instability.

6 Conclusion and Future Research

In our work, we represent 4 international exchanges as individual networks of partially correlated stock returns. To do so, we build a *Graph*, comprised of a set of *Vertices* and *Edges*, via the implementation of the joint sparse regression estimation techniques of Peng et. al (2009). This approach allows us to uncover some of the hidden topological features of a series of Markowitz *tangency portfolios*. We generally find that investing according to MPT dissuades the inclusion of highly central stocks in an optimally designed portfolio, hence keeping portfolio variances under control. We find that this result is market-dependent and more prevalent for certain countries than for others. From this cross-sectional network analysis, we learn that the interaction between a stock's *individual performance* (Sharpe ratio)

and *systemic performance* (eigenvector centrality) can be complex. This motivates us to explore the time-varying correlation ρ between Sharpe ratio and eigencentrality.

From our analysis, we find that ρ is both time and market-dependent, fluctuating between positive and negative values and taking values above and below our pre-defined threshold of 0.2. When $\rho <$ 0, the peripheral stocks in the network (with the smallest partial correlations) have the highest Sharpe ratios, and so this is where wealth should be allocated for optimal portfolio diversification benefits. In restricting investment to a subset of peripheral stocks, high individual stock performance is achieved with low portfolio risk. When $\rho > 0$, the stocks with the highest Sharpe ratios are those that are the most central. Hence, a trade-off emerges between investing in stocks with a high individual performance and increasing the systemic risk of the portfolio. Therefore, we should invest in the most central stocks only when the correlation ρ is a above a pre-determined threshold ($\tilde{\rho} = 0.2$). We decide to restrict our portfolios to a subset of 20 stocks based on the findings of Desmoulins-Lebeault and Kharoubi-Rakotomala (2012), who show that this number is optimal for portfolio diversification.

Based on the above, we implement and evaluate 4 investment strategies following an out-of-sample approach. Firstly, we introduce a *Naïve* portfolio where we invest in the targeted group of stocks via applying a simple 1/N diversification rule. We also design a *Tangency* strategy that invests in the targeted stocks weighted optimally by Markowitz's tangency portfolio. We then carry out the same procedure with the addition of short-sale constraints, and name this porfolio *Tang. Lim.* These 3 strategies are referred to as ρ-dependent. Finally, we introduce a benchmark strategy, *Market*, that is a proxy for investing in the entire market with allocation according to a 1/N diversification rule. This strategy is the benchmark to beat, as we are interested in ascertaining whether estimating the time-varying nature of network effects can lead to a portfolio simplification that enhances out-of-sample performance. We also introduce a *reverse-ρ* control procedure that reverses the criteria for investing in the targeted set of stocks, ruling out the possibility that any results that we obtain are achieved by chance.

The implementation of the above strategies provides a series of informative results. Firstly, we find that a ρ-dependent *Naïve* investment strategy is significantly ineffective in delivering superior out-of-sample performance in comparison to the bechmark. This finding is at odds with that of Peralta & Zareei (2016). We show that the two ρ-dependent Markowitz allocation strategies can indeed significantly enhance out-of-sample portfolio performance when compared to the *Market* benchmark. This contrasts the findings of DeMiguel et. al (2009), who show that the opposite result is true. Furthermore, we show that implementing a ρ-dependent method in conjunction with Markowitz allocation can lead to portfolios that are resilient to major macroeconomic instability, primarily to that of the recent 2008 Financial Crisis.

It is also important to highlight a number of limitations in our work. Firstly, the joint sparse estimation regression approach (SPACE) does not estimate K exactly, but only its sparsity pattern. It also does not fully exploit all of the information in the system, and the procedure does not ensure that K is always positive definite (Pourahmadi, 2013). Furthermore, in our analysis we only consider stocks that are active over the entire time series. This leads to a potential bias, as we omit the effect of any new stocks that enter the market after 01/01/2001. As shown by Peralta & Zareei (2016), we are hence left with stocks that are likely to be more *stable*, meaning that we may have a higher proportion of central stocks in our dataset. Finally, the major weakness of our approach is the selection of the threshold $\tilde{\rho}$, as

it is extremely difficult to find a value that is optimal.

Overall, we show that in considering the time-varying nature of partially correlated networks, we can enhance out-of-sample performance by simplifying the portfolio selection process and investing in a targeted subset of stocks. We also find that our work proposes a number of future research questions. Although we implement short-sale constraints, it would also be wise to introduce limits on the amount of wealth that can go into purchasing stocks, as this would help to avoid large portfolio variances. Furthermore, our work paves the way for future research into the ability of ρ-dependent investment strategies to enhance portfolio performance in times of macroeconomic distress and major financial crises.

REFERENCES

Back, K., 2017. Asset pricing and portfolio choice theory. *Oxford: Oxford University Press.*

Billio, M. et al., 2012. Econometric measures of connectedness and systemic risk in the finance and insurance sectors. *Journal of Financial Economics*, 104(3), pp.535–559.

Bonacich, P., 1972. Factoring and weighting approaches to status scores and clique identification. *The Journal of Mathematical Sociology*, 2(1), pp.113–120.

DeMiguel, V., Garlappi, L. & Uppal, R., 2009. Optimal Versus Naive Diversification: How Inefficient is the 1/N Portfolio Strategy? *Review of Financial Studies*, 22(5), pp.1915–1953.

Dempster, A., 1972. Covariance Selection. *Biometrics*, 28(1), pp.157-175.

Desmoulins-Lebeault, F. & Kharoubi-Rakotomalala, C., 2012. Non-Gaussian diversification: When size matters. *Journal of Banking & Finance*, 36(7), pp.1987–1996.

Diebold, F.X. & Yılmaz, K., 2014. On the network topology of variance decompositions: Measuring the connectedness of financial firms. *Journal of Econometrics*, 182(1), pp.119–134.

Frobenius, G., 1912. Ueber Matrizen aus nicht negativen Elementen. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften*, 26, pp.456—477.

Hastie, T., Tibshirani, R. and Wainwright, M. (2015). Statistical learning with sparsity. Boca Raton: CRC Press.

Hautsch, N., Schaumburg, J. & Schienle, M., 2014. Financial Network Systemic Risk Contributions. *Review of Finance.*

Landau, E., 1895. Zur relativen Wertbemessung der Turnierresultate. *Deutsches Wochenschach* 11, pp.366–369.

Markowitz, H., 1952. Portfolio Selection. *The Journal of Finance*, 7(1), pp.77-91.

Peng, J., Wang, P., Zhou, N. and Zhu, J., 2009. Partial Correlation Estimation by Joint Sparse Regression Models. *Journal of the American Statistical Association*, 104(486), pp.735-746.

Peralta, G. and Zareei, A., 2016. A network approach to portfolio selection. *Journal of Empirical Finance*, 38, pp.157-180.

Perron, O., 1907. Zur Theorie der Matrices. *Mathematische Annalen*, 64(2), pp.248–263.

Pourahmadi, M., 2013. High-dimensional covariance estimation. *Hoboken, New Jersey: Wiley*, pp.121-140.

Pozzi, F., Di Matteo, T. & Aste, T., 2013. Spread of risk across financial markets: better to invest in the peripheries. *Scientific reports*, 3, p.1665.

Apendix

UK

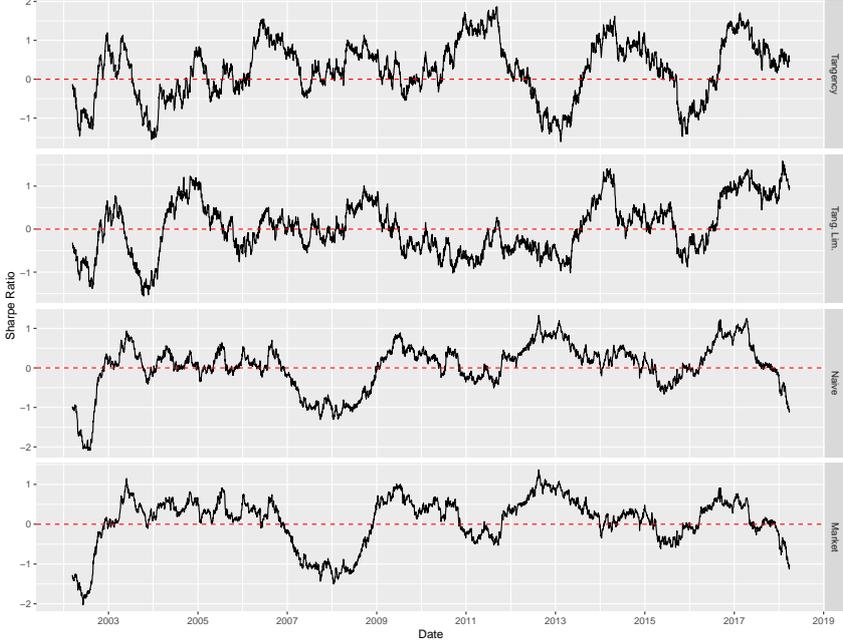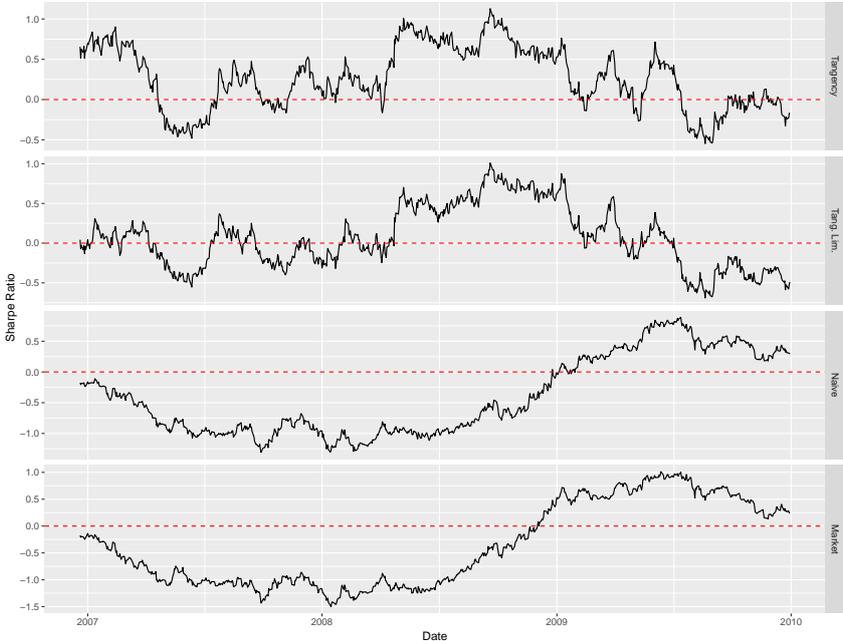Figure 5: UK 12-month Rolling Sharpe Ratios per Strategy.

Figure 6: UK 12-month Rolling Sharpe Ratios 2006-2009.

Germany

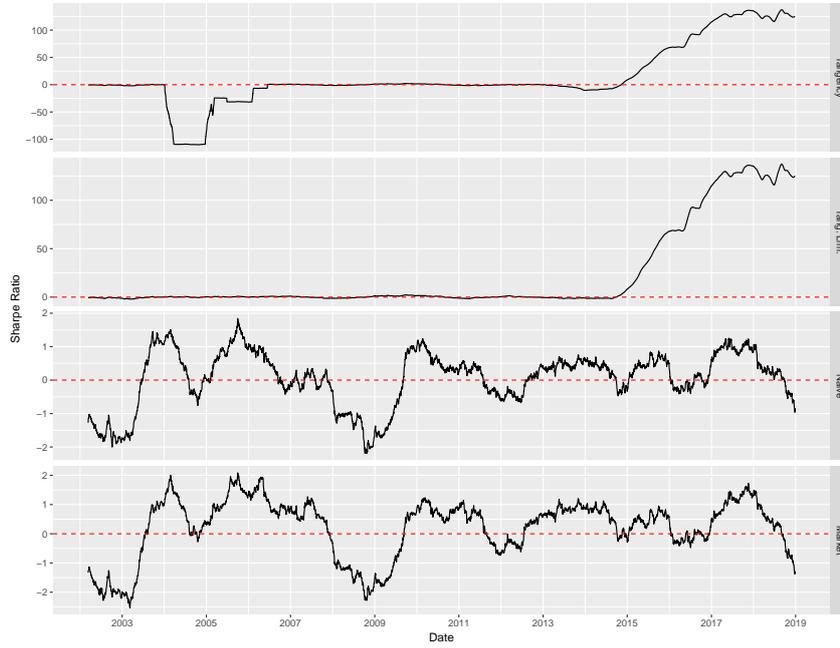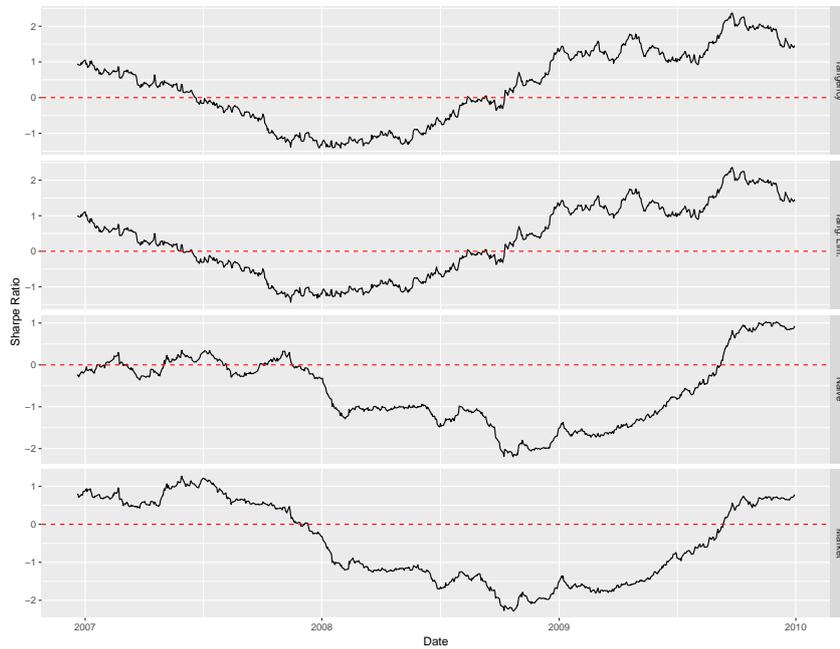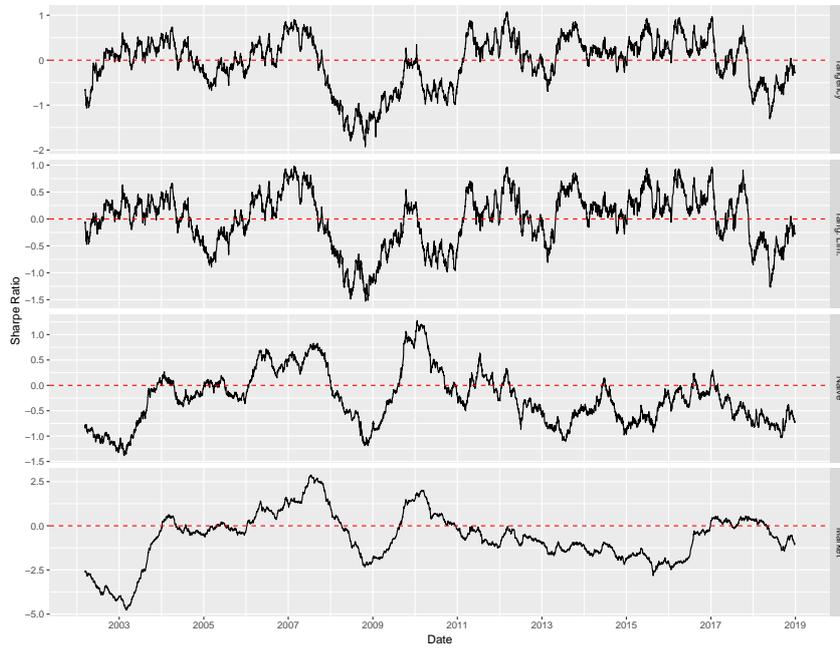Figure 7: Germany 12-month Rolling Sharpe Ratios per Strategy.

Figure 8: Germany 12-month Rolling Sharpe Ratios 2006-2009.

Brazil

Figure 9: Brazil 12-month Rolling Sharpe Ratios per Strategy.
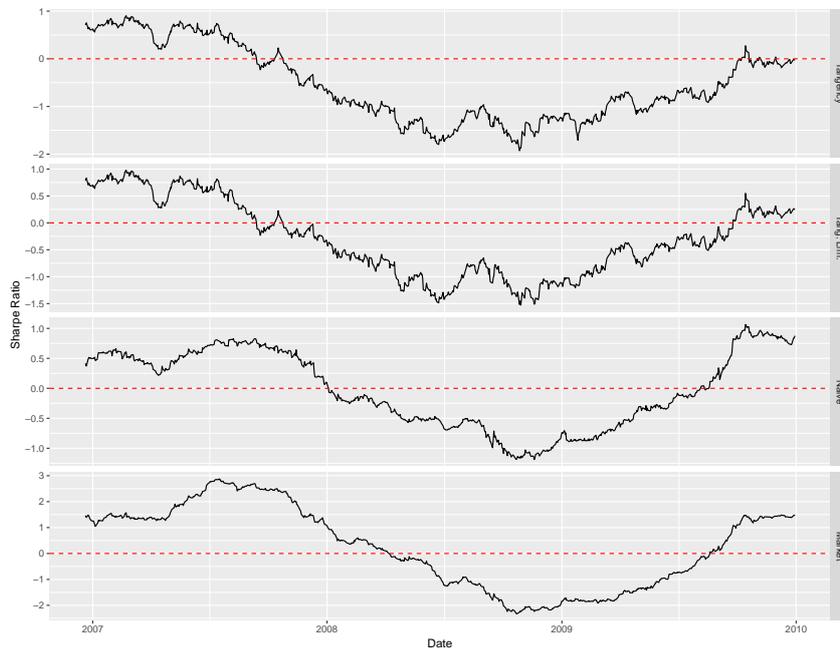

Figure 10: Brazil 12-month Rolling Sharpe Ratios 2006-2009.

India
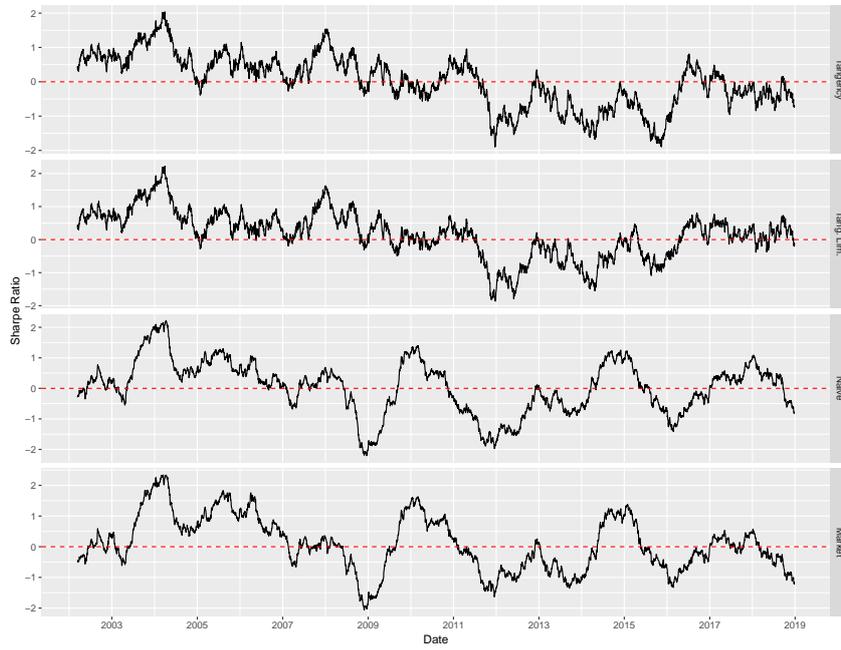
Figure 11: India 12-month Rolling Sharpe Ratios per Strategy.

Figure 12: India 12-month Rolling Sharpe Ratios 2000-2009.