Estimating Time-Varying Network Effects with Application to Portfolio Allocation

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Thesis Overview

Can estimating the time-varying topological features of a network lead to a portfolio simplification process that enhances out-of-sample performance?

- We characterize international financial markets as partially correlated networks of stock returns.
  - Mean-variance portfolios generally dissuade the inclusion of central stocks in the network.
  - Interaction of a stock's *individual* and *systemic* performance is complex.
  - Time-varying correlation of these features is highly market-dependent.

- We then implement investment strategies that allocate wealth to a targeted subset of stocks, contingent on the time-varying network dynamics.
  - Targeted mean-variance allocation shown to enhance out-of-sample performance.
  - Targeted 1/N allocation ineffective in enhancing out-of-sample performance.
  - Evidence that portfolios are resilient to periods of major macroeconomic instability.

Literature Review

- **DeMiguel et al. (2009)** evaluate the out-of-sample performance of Markowtiz mean-variance portfolios.
  - Naïve 1/N diversification rule outperforms MPT.
- **Peng et al. (2009)** design smart optimization shooting algorithm to estimate a sparse correlation matrix.
  - Joint sparse estimation regression, building on Neighborhood Selection.
- **Pozzi et al. (2013)** implement network-based investment strategies that improve portfolio performance.
  - Naïve 1/N allocation to stocks on the periphery of the network.
- **Peralta & Zareei (2016)** design investment strategies taking into account time-varying network features.
  - Target subset of stocks on network depending on time-varying network dynamics.

Data

- Study focused in both developed and emerging markets for stocks lised in:
  - **UK** (LSE), **Germany** (Deutsche Börse), **Brazil** (B3), **India** (NSE).

- Daily price data from **01/01/2001** to **31/12/2018**.
  - 120 most capitalized stocks.
  - Active over entire period.
  - **Thompson Reuters**.

- 3-month Treasury bill yields as proxy for “risk free” rate.
  - Converted to daily values.
  - For 4 different countries.
  - **Thompson Reuters**.

Building a Partial Correlation Network

Defining Partial Correlation

- Partial correlation measures the linear conditional dependence between two stocks y_{it} and y_{jt} controlling for the correlation of other variables in the system.
- Accounts for interference caused by confounding variables, removing noisy correlations with variables of interest: applicable to financial data.

$$\rho^{ij} = Corr\big(y_{it}, y_{jt} \big| \{y_{kt} : k \neq i, j\}\big) \tag{1}$$

Joint Sparse Regression Model (SPACE)

- When estimating partial correlation matrices with large amounts of data, we require a *shrinkage estimator* to create a *sparse* matrix of correlations.
- Peng et al. (2009) incorporate a LASSO-based joint sparse estimation technique with *absolute value penalty*.

$$\mathbf{min} \quad \sum_{i=1}^{n}\left[\sum_{t=1}^{T}\left(y_{it}-\sum_{j\neq i}^{n}\rho^{ij}\sqrt{\frac{\hat{k}_{jj}}{\hat{k}_{ii}}}y_{jt}\right)^2\right]+\lambda\sum_{i=2}^{n}\sum_{j=1}^{i-1}|\rho^{ij}| \tag{2}$$

- Where,
  - $K = \Sigma^{-1}$
  - $\lambda = 0.2 \times \mathrm{T}$

Computing Eigenvector Centrality

- Once we have built the partial correlation network, we are interested in the level of interconnectedness (*centrality*) of each node in the network.

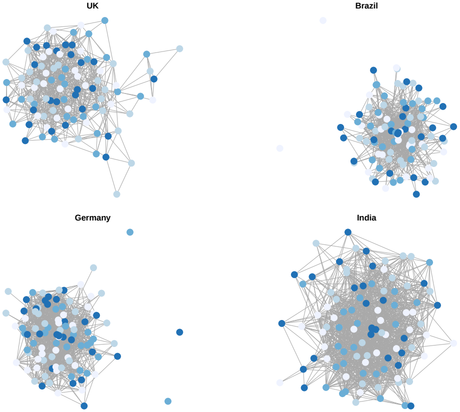
- As defined by Bonacich (1972), **eigenvector centrality** assumes that the centrality of a vertex i (v_i) is proportional to the weighted sum of the centralities of its neighbours (ν_j).

$$\nu_i \equiv \lambda^{-1} \Sigma_j \Omega_{ij} \nu_j \tag{3}$$

Exploring Modern Portfolio Theory Using Network Analysis

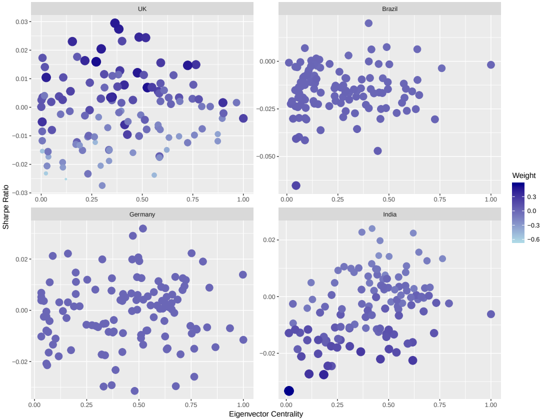
Tangency Portfolio as Partial Correlation Network

The Tangency Portfolio as a Partial Correlation Network.



Tangency Portfolio as Partial Correlation Network

Optimal Weights for Tangency Portfolio Strategy.



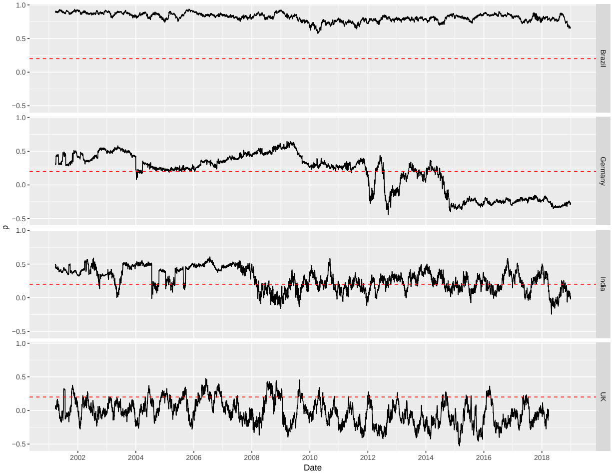
Implementing a $\rho-$Dependent Strategy

Defining ρ

- $\rho = corr(SR, eigencentrality)$, **individual** and **systemic** performance.
- $\rho \leq \tilde{\rho}$: wealth should be allocated to **least** central stocks.
- $\rho > \tilde{\rho}$: wealth should be allocated to **most** central stocks.
- $\tilde{\rho} = 0.2$: in keeping with the work of Peralta & Zareei (2016).

Visualizing the Time-Varying ρ

Time-Varying Correlation of Sharpe Ratio and Centrality (ρ).



Investment Strategies

- **The ρ-dependent strategies**
  - ***Tangency***: allocate wealth according to MPT's tangency portfolio on 20 selected stocks.
  - ***Tang. Lim***: same procedure as ***Tangency*** with short-sale constraints of 50% on selected stocks.
  - ***Naïve***: allocate wealth evenly (1/N) across 20 selected stocks.

- **The benchmark strategy**
  - ***Market***: allocate wealth evenly (1/N) across all stocks, acting as proxy for the market.

- **The reverse ρ-dependent strategy**
  - Reverses criteria for investing in 20 stocks for the ρ-dependent strategies.
  - Control strategy: to show results not achieved by chance.

Out-of-Sample Approach

- Calculate the at time t, the Sharpe ratio, centrality score and ρ for the period $[t - 60, t]$.
- Rank stocks according to centrality score and pick the 20 least or most central stocks according to ρ.
- For the 20 selected stocks: calculate $w*_t$ of each strategy and apply those weights to time $t + 1$.
- Repeat the process at every period time period (day).

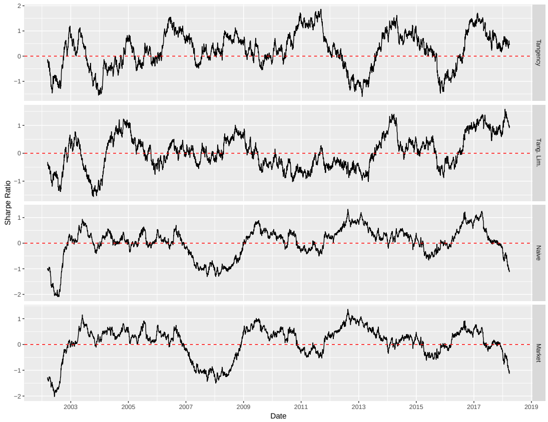
Results

Overview

Overall, we show that in considering the time-varying nature of partially correlated networks, we can enhance out-of-sample performance by simplifying the portfolio selection process and investing in a targeted subset of stocks.

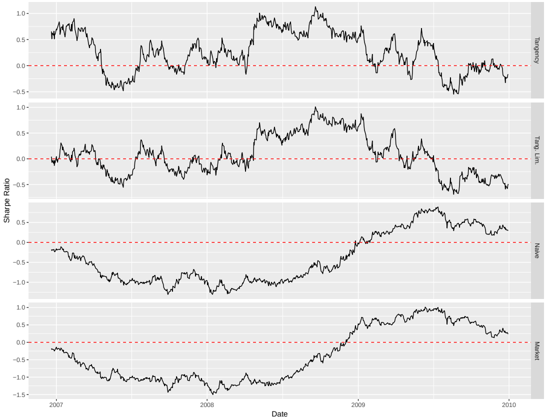
UK

UK 12-month Rolling Sharpe Ratios per Strategy.



UK: 2006-2009

UK 12-month Rolling Sharpe Ratios 2006-2009.



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UK

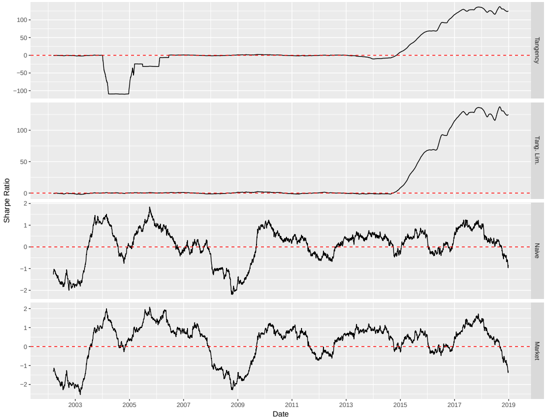
Table: UK 12-month Rolling Mean Sharpe Ratios.

Period & Strategy	Tangency	Tang. Lim	*Naïve*	Market
All sample ρ-strategy	0.2416***	0.0206	0.0340**	0.0780***
	(0.0153)	(0.0151)	(0.0151)	(0.0151)
2006-2009 ρ-strategy	0.2711***	0.0693**	-0.4008***	-0.3622***
	(0.0370)	(0.03641)	(0.0378)	(0.0375)
All sample reverse ρ	-0.7074***	0.2502***	0.1536***	0.0780***
	(0.0171)	(0.0155)	(0.0154)	(0.0151)
2006-2009 reverse ρ	0.5954***	0.6074***	-0.3590***	-0.3622***
	(0.0395)	(0.0395)	(0.0375)	(0.0375)

$*p < 0.10, **p < 0.05, ***p < 0.001, H_0 : SR = 0$

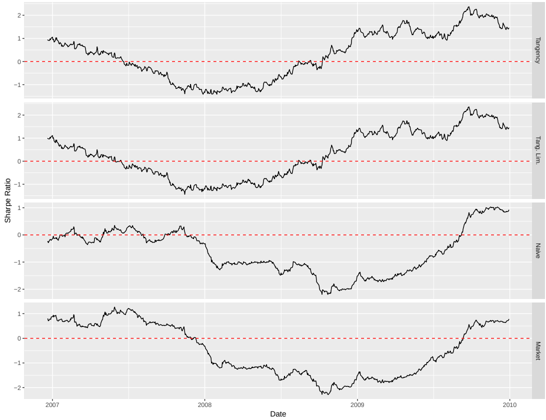
Germany

Germany 12-month Rolling Sharpe Ratios per Strategy.



Germany: 2006-2009

Germany 12-month Rolling Sharpe Ratios 2006-2009.



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Germany

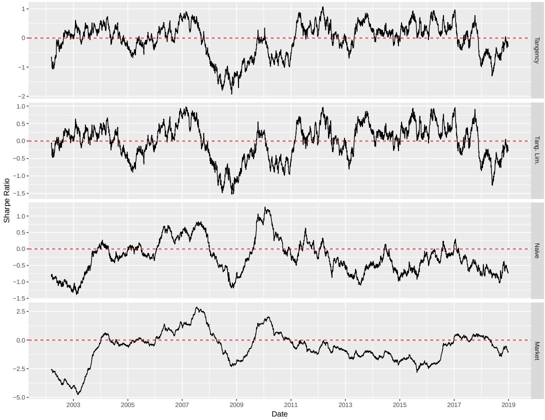
Table: Mean 12-month Rolling Sharpe Ratios.

Period Strategy	Tangency	Tang. Lim	*Naive*	Market
All sample ρ-strategy	13.8069***	22.4643***	0.0643***	0.2204***
	(0.1482)	(0.2403)	(0.0151)	(0.0152)
2006-2009 ρ-strategy	0.2935***	0.2824***	-0.6197***	-0.4863***
	(0.0371)	(0.0371)	(0.0397)	(0.0385)
All sample reverseρ	-107,6349***	-0.2485***	0.04386***	0.2204***
	(1.1660)	(0.0155)	(0.0153)	(0.0152)
2006-2009 reverse ρ	-298.8850***	-0.8142***	-0.6368***	-0.4863***
	(7.6866)	(0.0420)	(0.0399)	(0.0385)

$*p < 0.10, **p < 0.05, ***p < 0.001, H_0 : SR = 0$

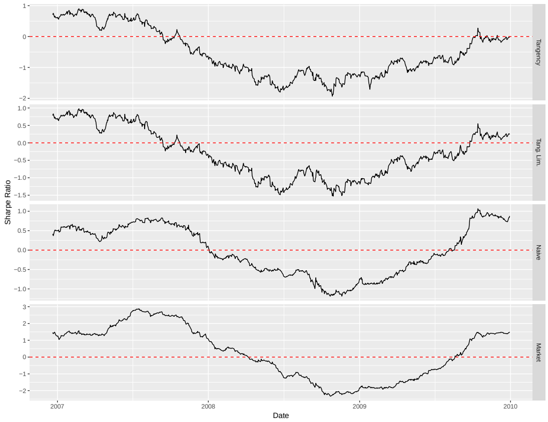
Brazil

Brazil 12-month Rolling Sharpe Ratios per Strategy.



Brazil: 2006-2009

Brazil 12-month Rolling Sharpe Ratios 2006-2009.



Brazil

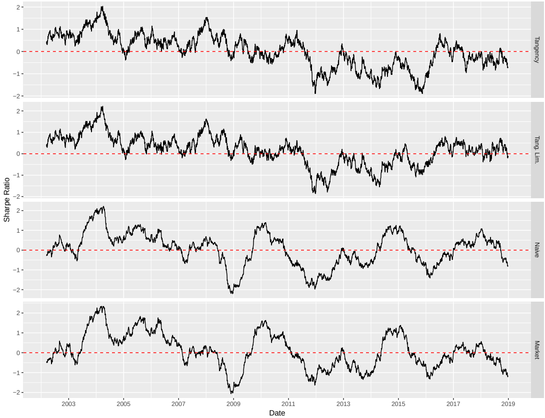
Table: Mean 12-month Rolling Sharpe Ratios.

Period Strategy	Tangency	Tang. Lim	*Naive*	Market
All sample ρ-strategy	-0.0474***	-0.0072	-0.2498***	-0.6503***
	(0.0151)	(0.0151)	(0.0153)	(0.0168)
2006-2009 ρ-strategy	-0.5401***	-0.3060***	-0.0184	0.2410***
	(0.0389)	(0.0372)	(0.0364	(0.0369)
All sample reverse ρ	-203.8342***	-1.1110***	-119.4047***	-0.6503***
	(2.19)	(0.0194	(1.2833)	(0.0168)
2006-2009 reverse ρ	-280.1428***	-0.3374***	-170.9398***	0.2410***
	(7.2000)	(0.0420)	(4.3960)	(0.0369)

$*p < 0.10, **p < 0.05, ***p < 0.001, H_0 : SR = 0$

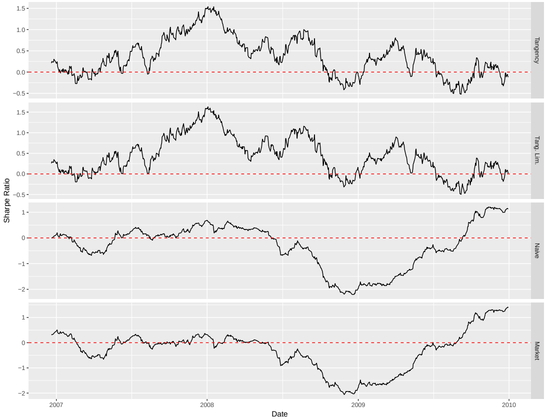
India

India 12-month Rolling Sharpe Ratios per Strategy.



India: 2006-2009

India 12-month Rolling Sharpe Ratios 2000-2009.



India

Table: Mean 12-month Rolling Sharpe Ratios.

Period Strategy	Tangency	Tang. Lim	*Naive*	Market
All sample ρ-strategy	0.0185	0.1635***	0.0469***	0.0970***
	(0.0151)	(0.0152)	(0.0151)	(0.0151)
2006-2009 ρ-strategy	0.3620***	0.4355***	-0.3082***	-0.2962***
	(0.0375)	(0.0380)	(0.0372)	(0.0371)
All sample reverse ρ	-61.5554***	-0.4624***	0.1473***	0.0970***
	(0.6580)	(0.0160)	(0.0153)	(0.0151)
2006-2009 reverse ρ	-60.3890***	-0.5256***	-0.2215***	-0.2962***
	(1.5435)	(0.0388)	(0.0368)	(0.0371)

$*p < 0.10, **p < 0.05, ***p < 0.001, H_0 : SR = 0$

Conclusion and Future Research

Conclusion

- Investing according to MPT dissuades the inclusion of highly central stocks, hence keeping portfolio variances under control. However, it is market dependent.
- Stock's individual performance and systemic performance can be complex. We find that the relationship is time and market dependent.
- This motivates the analysis of the time varying corelation ρ, and invest accordingly.
- Based on the above, we implement and evaluate 3 ρ-dependent investment strategies following an out-of-sample approach.

Conclusion

- **ρ-dependent *Naïve* strategy:**
    - Significantly ineffective in delivering superior out-of-sample performance compared to the benchmark.
    - This finding is at odds with that of Peralta & Zareei (2016).

- **Markowitz ρ-dependent strategies:**
    - The strategies can significantly enhance out-of-sample performance whem compared to the benchmark.
    - Markowitz ρ-dependent strategy can lead to portfolios that are resilient against major macroeconomic instability.
    - Tangency Limited portfolio can protect against large fluctuations in returns.

Future Research

- Method of selection of the threshold $\tilde{\rho}$, for each market and time period.
- Adapting research to include *all* stocks over the period, whether IPO or delisted.
- Implement regulatory-dependent long and short constraints to the Markowitz ρ-dependent portfolios.
- Ability of the ρ-dependent investment strategies to enhance portfolio performances in times of macroeconomic distress, by analyzing periods other the 2008 Financial Cirses.

Thank You

Appendix

$$y_{it} = \theta_0 + \Sigma_{i \neq j} \theta_{ij} y_{jt} + u_i \tag{4}$$

$$\theta^{ij} = -\frac{k^{ij}}{k^{ii}} = \rho^{ij} \sqrt{\frac{k^{ii}}{k^{jj}}} \tag{5}$$

$$\rho^{ij} = -\frac{k^{ij}}{\sqrt{k^{ii} k^{jj}}} \tag{6}$$

$$\textbf{min} \quad \sum_{i=1}^{n} \left[\sum_{t=1}^{T} \left(y_{it} - \sum_{j \neq i}^{n} \rho^{ij} \sqrt{\frac{\hat{k}_{jj}}{\hat{k}_{ii}}} y_{jt} \right)^2 \right] + \lambda \sum_{i=2}^{n} \sum_{j=1}^{i-1} |\rho^{ij}| \tag{7}$$