MASTER PROJECT

Pierre Coster
Pia Ennuschat
Raquel Lorenzo
Giacomo Stazi
Robert Wojciechowski

Barcelona Graduate School of Economics

Master in Economics

Can adjustment costs of intangible capital explain the decline in the labor share?

Supervisor Prof. Manuel Santana

June 2020

Abstract

Labor share, once thought to be a constant, has experienced a secular decline in many developed economies. We investigate whether adjustment costs to intangible capital can be used to explain this trend. We develop a simple partial equilibrium model with a profit maximizing firm that produces using a three factor CES production function and faces convex adjustment costs to intangible capital. We find an intuitive expression for the steady state labor share as a function of parameters and the steady state level of investment in intangible capital. We then run simulations to better understand the behaviour of the labor share in our model. Somewhat surprisingly, we find that adjustment costs do not affect the steady state labor share for any given elasticity of substitution. However, their presence creates a strong relationship between the labor share and the elasticity of substitution. We also find a number of short-run dynamics that are affected by the level of adjustment costs.

Acknowledgements

We are grateful to Manuel Santana, Marta Morazzoni and Andrea Chiavari for their advice and support throughout this research. We are also thankful to Isaac Bailey for his teaching and insights on the concept of adjustment costs. Any remaining errors are our own.

Table of Contents

List of Figures

List of Tables

1 Introduction

Initially believed to be constant (Kaldor, 1955), the labor share of GDP has fallen significantly in recent decades in advanced economies. Defined as total nominal wages over total nominal GDP, it measures the share of national income that is allocated to labor, and hence to capital. Given the methodological variations between national statistical offices, the harmonised series of AMECOs are the most widely used in the literature and allow a robust comparison between countries (Schneider, 2011), which we observe in Figure 1.1. Most countries experienced a long decline in their labor share between 1980 and the mid-2000's. A more heterogeneous evolution then followed, with some countries continuing their fall, such as the United States and Spain, while other countries have levelled off their decline, or even recovered substantially, such as the United Kingdom. Not shown in the figure, but of similar importance, the decline in the labor share is experienced by virtually all sectors (Karabarbounis and Neiman, 2013).

This has important implications, both in terms of public policy and academic research. As regards to economic policy, the decline in the labor share can be more easily interpreted as a decoupling of labor productivity from wages. This raises concerns of sluggish wage growth and weakened purchasing power. This is particularly prevalent for firms below the technological frontier, and has important implications for skills and labor market policies. Moreover, an increase in the share of capital income at the expense of labor income has possible consequences on wealth inequality (OECD, 2018).

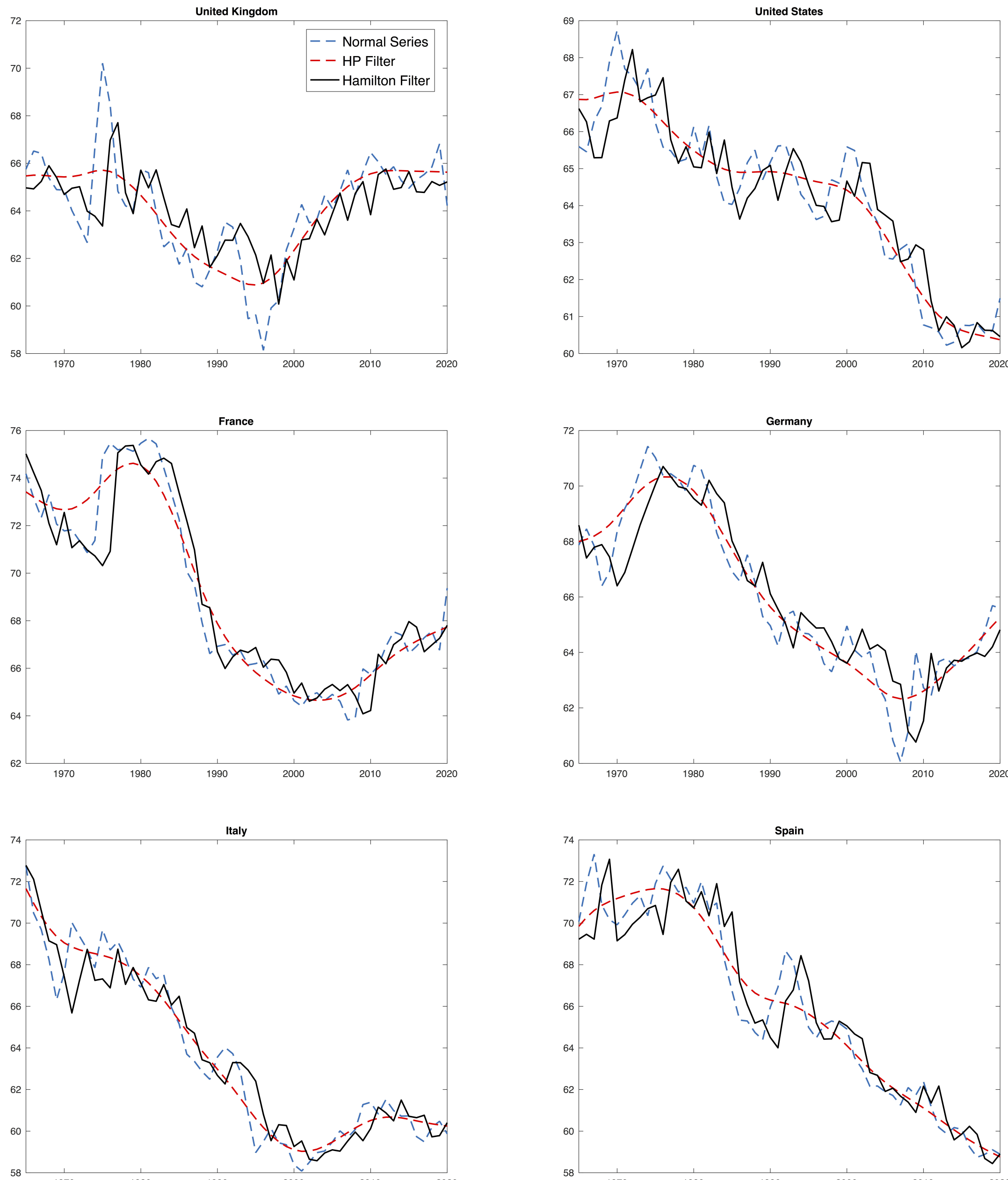
As for research, five main explanations (some conflicting, some intertwined) have been identified: (i) globalization, (ii) the declining unionization of workers, (iii) the rise of superstar companies, (iv) the emergence of new technologies, and (v) the substantial growth in real estate prices. Overall, explanations (iii) and (iv) are those that receive greater consensus in the literature (International Monetary Fund, 2017; McKinsey Global Institute, 2019).

First of all, trade integration is found to put a downward pressure on the labor share of income in advanced economies, through the offshoring of labor-intensive tasks and through import competition (International Monetary Fund, 2017). To a lesser extent, but still significantly, the decline in unionization is pushing down the labor share (International Monetary Fund, 2017; Sahin, Elsby and Hobijn, 2013). A third explanation is the increasing market share of superstar companies, with low labor ratios, which is critical in the fall of the labor share of income (Autor et al., 2017). Last, but not least, the change in the composition of capital plays a key role. The increased use of labor-substitutable technologies, such as robots or ICTs, is weighing down on employment and wages, the two numerators of the labor share equation (see equation 2.11) (Acemoglu and Restrepo, 2017). In the same vein, the fall in the relative price of these technologies could explain up to half of the decline in the global labor share (Karabarbounis and Neiman, 2013). Finally, some research finds that the increase in the share of capital in total income is almost exclusively explained by the rising cost of real estate, thereby challenging previous findings (Rognlie, 2015). Some of the more recent research even suggests that the share of labor in total income is only a statistical artifact, and that by correcting for some methodological biases, this share is roughly constant over time (Koh, Santaeulàlia-Llopis and Zheng, 2016; Cette, Koehl and Philippon, 2019).

These explanations, which are complementary for some and contradictory for others, call for more research into the causes of the fall in the labor share of income. In this thesis, we will focus on explanation (iv), namely the rise of intangible capital as a factor of production. A crucial feature of this intangible capital is that it entails higher adjustment costs than physical capital. Investment adjustment costs are the costs that companies face when investing, i.e. in addition to the price of the investment itself. Heuristically, when companies invest, capital must be in place and ready to produce. This requires time and money, often referred to as "time to build" and "adjustment costs". Some inputs will not be available to produce during the installation process, resulting in lost earnings. The research on the link between these adjustment costs and macroeconomic trends is in its nascent stages (Chiavari and Goraya, 2020). A full understanding of these phenomenon has not yet been achieved, and we believe there is room for further research. In light of this, we are contributing to this literature by proposing a model that links the adjustment costs of intangible capital to the labor share of income.

The remainder of this thesis is organized as follows. In section 2, we derive an equation for the labor share from a partial equilibrium perspective. We go on by describing some *ceteris paribus* steady state dynamics. In section 3, we run simulations in order to identify steady state as well as short run dynamics. We conclude with a summary of our findings in section 4.

Figure 1.1: Labor Share trends over the past 60 years in advanced economies



Source: AMECO

Note: Adjusted labor share with GDP at current factor cost (minus taxes and plus subsidies), in line with previous literature (Guerriero, 2019). The HP filter uses $\lambda = 100$ (Hodrick and Prescott, 1997), while the Hamilton filter uses $h = 2$ (Hamilton, 2018).

2 The Model

In this section we present a partial equilibrium model of investment dynamics in which a representative firm employs three production factors; labor, physical and intangible capital. We assume that markets are competitive, which implies that the firm takes the prices of labor and the two types of capital as given. Moreover, since we focus on a partial equilibrium model, all prices are exogenous.

The firm hires workers and rents physical capital in every period without incurring additional costs or frictions. This implies that the problem of choosing the optimal amount of labor and physical capital is static. Intangible capital is purchased and accumulated by the company, and these investment entail additional adjustment costs. In particular, we introduce convex adjustment costs through a quadratic function of investment.

Since we want to study the evolution of factor shares over time, we specify a CES technology. Indeed, if markets are competitive and firms operate using a Cobb-Doubglas production function, then capital and labor shares are entirely determined by technology. In other words, economic behaviours (e.g. demand fluctuations, elasticity of substitution) are irrelevant and factor shares are constant over time, typically set to two thirds for the labor share and to one third for the capital share (Growiec, McAdam and Mućk, 2018). In contrast, a CES production function allows factor shares to depend on the elasticity of substitution between factor of productions and on economic variables in general.

2.1 Setup and Solution

Consider a firm which maximizes the expected present value of its net revenues:

$$\max V_t = E_0 \left[\sum_{t=0}^{\infty} \left(\frac{1}{R} \right)^t \Pi_t \right] \tag{2.1}$$

$$\Pi_t = \Pi[K_t, S_t, L_t] = F(K_t, L_t, I_t) - p_t I_t - C(I_t) - wL_t - RK_t \tag{2.2}$$

$$p_t = \mu(1-\rho) + \rho p_{t-1} + \epsilon_t; \quad \text{where}\, \epsilon_t \sim \mathcal{N}(0,1). \tag{2.3}$$

where Π_t are the firm's profits in each period, K_t, L_t, S_t denote physical capital, labor and intangible capital, I_t is investment in intangible capital, w, p_t, R represent wages, the price of intangible capital and the rental rate of physical capital, and $C(I_t)$ is the cost of adjusting intangible capital. Prices are exogenously determined. In particular, we assume that the price of intangible capital follows an autoregressive model of order one, with coefficient $\rho < 1$ and mean μ. Note that μ is also the steady state value of prices.

The production technology is given by a three factor CES, and is subject to the following constraints on investment:

$$Y_t = F(K_t, L_t, I_t) = \left(a_K K_t^{\frac{\sigma-1}{\sigma}} + a_S S_t^{\frac{\sigma-1}{\sigma}} + a_L L_t^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{2.4}$$

$$I_t = S_t - (1-\delta) S_{t-1} \tag{2.5}$$

$$C(I_t) = \frac{\gamma}{2} I_t^2 \tag{2.6}$$

The problem of the firm consists on choosing the path of labor, physical and intangible capital in order to maximize the expected stream of net revenues subject to the constraints in 2.5 and 2.6. Note that the problem of hiring labor and acquiring physical capital is static. The use of both factor of productions can be chosen in every single period. On the other hand, investment in intangible capital involves a dynamic problem. To simplify the algebra we assume that there is *no time to build* for intangible capital, i.e. investment in intangible capital becomes productive in the same period that the investment was made.

Using standard dynamic programming techniques, we define the value function as:

$$V_t(S_{t-1}, p_t) = \max_{S_t, L_t, K_t} \left[\Pi_t(K_t, S_t, L_t) + \left(\frac{1}{R}\right) E_t\left[V_{t+1}(S_t, p_{t+1})\right]\right] \tag{2.7}$$

The first order conditions with respect to K_t, L_t and S_t are given by:

$$\left(\frac{Y_t}{K_t}\right)^{\frac{1}{\sigma}} a_K = R \tag{2.8}$$

$$\left(\frac{Y_t}{L_t}\right)^{\frac{1}{\sigma}} a_L = w \tag{2.9}$$

$$I_t = \frac{1}{\gamma}\left[\left(\frac{Y_t}{S_t}\right)^{\frac{1}{\sigma}} a_S - p_t + \frac{1-\delta}{R} E_t[p_{t+1}]\right] + \frac{1-\delta}{R} E_t[I_{t+1}] \tag{2.10}$$

As in any competitive firm problem, the rental rate of capital as well as wage equate to their marginal product. Due to the frictions we introduced for investment in intangible capital, we see from equation 2.10 that investment depends negatively on the user cost ($UC = p_t - \frac{1-\delta}{R} E_t\left[p_{t+1}\right]$) of intangible's investment. The user cost is defined as the unit cost of investment for one period, and is an estimate of the rental rate of intangible capital. In addition, investment depends positively on $F_{S_t} = \left(\frac{Y_t}{S_t}\right)^{\frac{1}{\sigma}} a_S$. In a frictionless model, $UC = F_{S_t}$. This is not our case, and hence the optimal level investment could differ from the one in a frictionless model.

2.2 The Labor Share

We define the labor share as follows:

$$(LS)_t = \frac{wL_t}{Y_t} \tag{2.11}$$

Using the first order conditions (see 5 Appendix), we obtain:

$$\boxed{(LS)_t = 1 - a_S^\sigma \left[\gamma\left(I_t - \frac{1-\delta}{R}E_t\left[I_{t+1}\right]\right) + p_t - \frac{1-\delta}{R}E_t\left[p_{t+1}\right]\right]^{1-\sigma} - a_K^\sigma R^{1-\sigma}} \tag{2.12}$$

Equation 2.12 depicts the the evolution of the labor share as a function of the elasticity of substitution σ, the adjustment cost parameter γ, intangible and physical capital's factor shares a_S and a_K, physical capital's rental rate R, the expected evolution of intagible's price p_t and of investment I_t, and depreciation δ.

Evaluated at the steady state level of investment and at steady state prices ($p_t = \mu$), labor share is:

$$\boxed{(LS)^* = 1 - a_S^\sigma \left[(\gamma I^* + \mu)\left(\frac{R-1+\delta}{R}\right)\right]^{1-\sigma} - a_K^\sigma R^{1-\sigma}} \tag{2.13}$$

With μ being the steady state price (see 2.3) and $I^* = S^*\delta$ the steady state investment.

It is important to note that in the analysis of the following two subsections (2.2.1 and 2.2.2), all the intuitions are valid only if we keep all variable as parameters. In other words, the analysis is done all other things being equal: *ceteris paribus*.

2.2.1 Partial equilibrium analysis of the labor share, *ceteris paribus*

We will start with a partial equilibrium analysis of equation 2.12. Again, it is important to note that the following analysis is correct only when keeping all other parameters and variables constant (*ceteris paribus*). In this context, it has some particularly key features:

- When $\sigma = 1$:

It is the **Cobb-Douglas case** and the labor share is constant: $(LS)_t = 1 - a_S - a_K$.

- When $\sigma > 1$:

This is the case where **inputs are substitutes**. All things being equal, the labor share **increases** with an increase in γ (provided that $\left(I_t - \frac{1-\delta}{R} E_t\left[I_{t+1}\right]\right) > 0$). The labor share also **increases** with an expected decrease in I_{t+1} or p_{t+1}, and an increase in I_t or in p_t. All things being equal, the labor share **decreases** when these same variables move in the opposite direction than the one mentioned above.

- When $\sigma < 1$:

This is the case where **inputs are complements**, and the effects of the different variables on the labor share are the opposite of those described in the $\sigma > 1$ case.

It is however important to note that this static analysis is limited in the sense that equation 2.12 depends on the endogenous evolution of the intangible capital stock S_t. This equation also depends on exogenous prices, which themselves affect these steady state values. In order to further our analysis and partly address this limitation, we will study the labor share at its steady state, i.e equation 2.13.

2.2.2 Pseudo steady state analysis of the labor share, *ceteris paribus*

We will now focus on equation 2.13. Once again, the following analysis and the according figures are a *pseudo* steady state analysis. In fact, all the intuitions are valid only if we treat steady state investment (I^*) as a parameter, and run our analysis *ceteris paribus*. Having this in mind:

- When $\sigma > 1$:

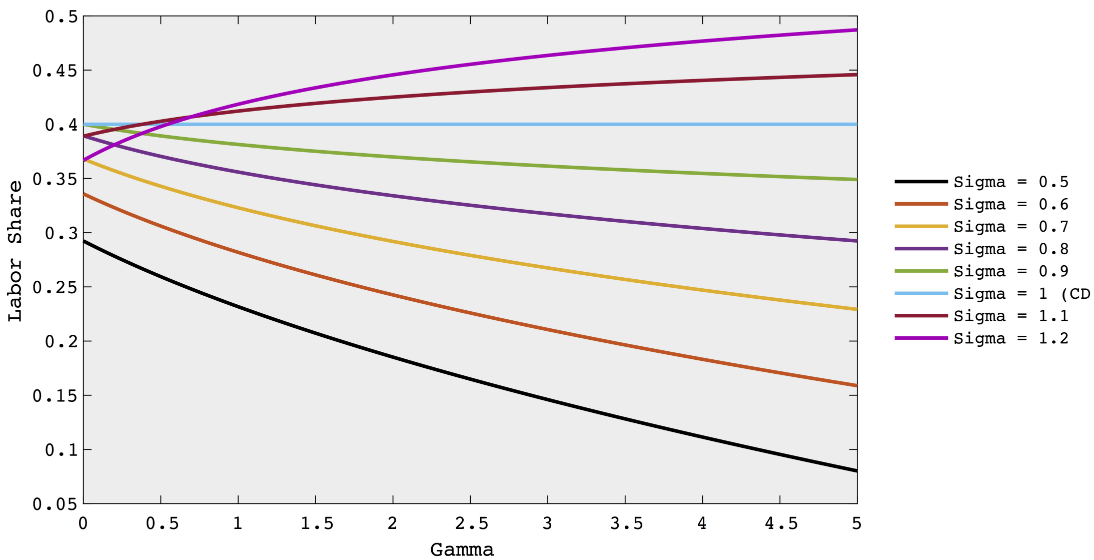
This is the case where **inputs are substitutes**. All things being equal, the labor share **increases** with an increase in γ, I^* or μ, while it **decreases** with a decrease of these same variables. In particular, the fact that a decrease in the steady state price of intangible leads to a decrease in the labor share is in line with the literature (Karabarbounis and Neiman, 2013).

- When $\sigma < 1$:

This is the case where **inputs are complements**, and the effects of the different variables on the level of capital are the opposite of those described above.

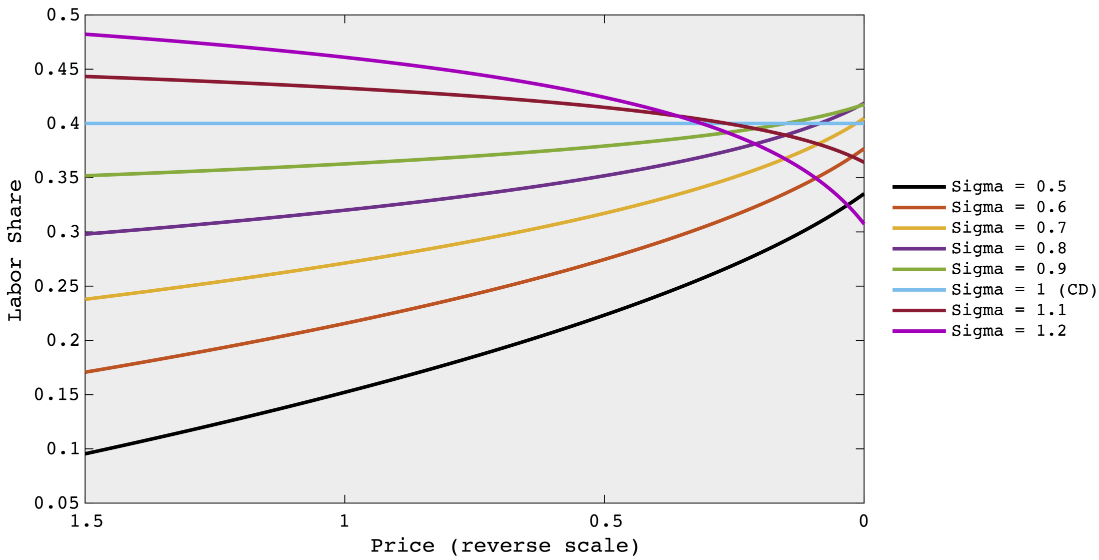
These pseudo steady state dynamics, where I^* is kept constant (not influenced by the level of γ), can be seen in Figure 2.1.

Figure 2.1: Pseudo steady state labor share as a function of the adjustment cost γ



a_S	a_K	R	S^*	δ	I^*	μ
0.3	0.3	1.05	1.5	0.2	0.3	0.3

Figure 2.2: Pseudo steady state labor share as a function of μ



a_S	a_K	R	S^*	δ	I^*	γ
0.3	0.3	1.05	1.5	0.2	0.3	0.5

Figure 2.1 and 2.2 depict the pseudo steady state evolution of the labor share depending on the value of the adjustment cost gamma and on investment's price, for different elasticities of substitution sigma. It graphically retranscribes the previously discussed intuitions. The parametrization used can be found below the figures.

Nevertheless, as previously explained, some of the variables we have defined as parameters and kept constant affect each other (for instance μ and I^*). This undermines our interpretation. For a more complete analysis of our model, we must resort to a dynamic simulation using Dynare.

3 Simulations

We have estimated our model by simulating a random sample of 50,000 observations and using an exogenous random process for modeling price in the form of an AR(1):

$$p_t = \mu(1-\rho) + \rho p_{t-1} + \epsilon_t; \quad \text{where } \epsilon_t \sim \mathcal{N}(0,1)$$

The use of a stochastic price allows us to assess, not only the effects of changes in adjustment costs on steady state levels of the economic variables; but also, the interaction between adjustment costs and shocks to prices. It has been argued that labor share decline could be partly explained by a decline in the relative price of intangible capital (Karabarbounis and Neiman, 2013). Therefore, the role of adjustment costs as amplifiers or mitigators of changes in prices is also central to our research question.

3.1 Baseline Parametrization

It is important to note that the goal of this part of the project is to use Dynare as a tool to analyse the relationships between labor share and the key parameters in our model. For this purpose we picked a baseline parametrization that was computationally robust to changes in the key parameters of interest, rather than the most realistic parametrization we could think of. Having said that, we have ensured that none of the findings that follow rely on the values we have assigned to any particular parameter. For the sake of brevity, we will not discuss the values of every parameter and instead focus on the key parameters of interest.

Due to different calibration methods and data sets, a wide range of values for γ has been found by the literature. Using quadratic convex adjustment costs similar to ours, estimates range from 0.455 (Cooper and Haltiwanger, 2006) to 20 (Hayashi, 1982). We will thus use a value of γ equal to 0.5 as a baseline, but we have investigated the labor share for a range of values between 0.2 and 15, and for different values of σ.

The elasticity of substitution is another parameter of interest that we often change in the analysis that follows. We often select values of σ quite far away from 1 as it allows us to compare how the model behaves when the factors are easily substitutable compared to when they are strong complements. Clearly, the case of $\sigma = 1$ corresponds to the Cobb-Douglas production function under which factor shares are constant, thus making it uninteresting for our analysis.

Since in our model the prices are fixed, they do not have any meaning other than in relation to each other. Therefore, we decided to set the prices of all factors to the same value of 0.3 (for the price of the investment good this is true in expectation). We have opted for quite a high volatility of the stochastic price process as we are interested in rigidities arising from convex adjustment costs and these effects are more important in the presence of large shocks (this is also investigated in the analysis that follows). Please note that the high rental rate of tangible capital ($r = 0.3$), can be justified by the fact that this rate has to compensate the renter for the depreciation of physical capital as well as the interest rate that prevails in the economy. Table 3.1 lists the parameter values of our baseline simulations. Some of them are going to change over the course of our analysis.

Table 3.1: Baseline parameters

Symbol	**Value**	**Parameter**
σ	1.10	Elasticity of substitution
a_K	0.25	Factor share - tangible capital
a_S	0.25	Factor share - intangible capital
a_L	0.50	Factor share - labor
δ	0.20	Deprecation rate of intangible capital
γ	0.50	Adjustement cost of intangible capital
r	0.30	Rental rate of tangible capital
R	1.30	Firm's discount rate
w	0.30	Wage
μ	0.30	Mean of the AR(1) price process
ρ	0.30	AR(1) persistance parameter of the price process
σ_p	0.10	Std deviation of the error of the stochastic process

3.2 Steady State Analysis

3.2.1 Adjustment costs, elasticities and the labor share

Analyzing different steady state values, i.e. under different levels of adjustment costs and elasticities of substitution, we find that a change in adjustment cost barely affects the steady state labor share, regardless of the level of substitutability between factors -see table 3.2. Only for very high levels of γ, e.g. $\gamma = 15$, do we observe a very slight increase in the labor share for both elasticities.

These results are in contrast with the predictions presented above in Section 2.2. The main driver of these differences is that when drawing the above conclusion, investment was set to be fixed. In the simulations, the steady state investment level is allowed to change given the different levels of adjustment costs.

The dynamics can be analysed through the original labor share equation 2.11, focusing on the changes of labor and output across the different steady state. As we take a partial equilibrium approach the wage level is taken as given. Alternatively, the dynamics can be explained through the equation 2.12, relating the steady state labor share to steady state investment.

When labor and intangible capital are substitutes, a reasonable expectation, as argued above, is that an increase in the cost of adjusting intangible capital leads firms to use labor and physical capital more extensively. Instead, independent of the substitutability of inputs, we find that both the steady state level of labor and output decrease as adjustment costs increase. This is a consequence of the firm's higher expenses caused by increased adjustment costs. No matter what is the substitutability between factors, as long as the firm uses some intangible capital $S_t > 0$, an increase in adjustment costs is going to make the firm "poorer". Thus, less able to purchase S_t , but simultaneously also less able to employ the other two factors.

Looking back at equation 2.13, we can relate the steady state labor share to steady state investment level. The intuition is more straightforward. For convenience, we rewrite it here, plugging in the value of investment in Steady State:

$$(LS)^* = 1 - a_S^\sigma \left[(\gamma\delta S^* + \mu) \left(\frac{R - 1 + \delta}{R} \right) \right]^{1-\sigma} - a_K^\sigma R^{1-\sigma}$$

In response to an increase in adjustment costs, the steady state level of intangible capital drops and, thus, investment level contracts - recall, $I^* = \delta S^*$. However, we just evidenced the labor share is indifferent to changes in γ.

Thereby, adjustment costs and steady state investment move in opposite directions in such a way that the effects exactly offset one another, keeping labor share unchanged. Intuitively, when adjustment costs increase, they effectively make intangible capital more costly compared to other factors. Therefore it is unsurprising that higher adjustment costs lower the steady state level of intangible capital and by consequence also the steady state level of investment.

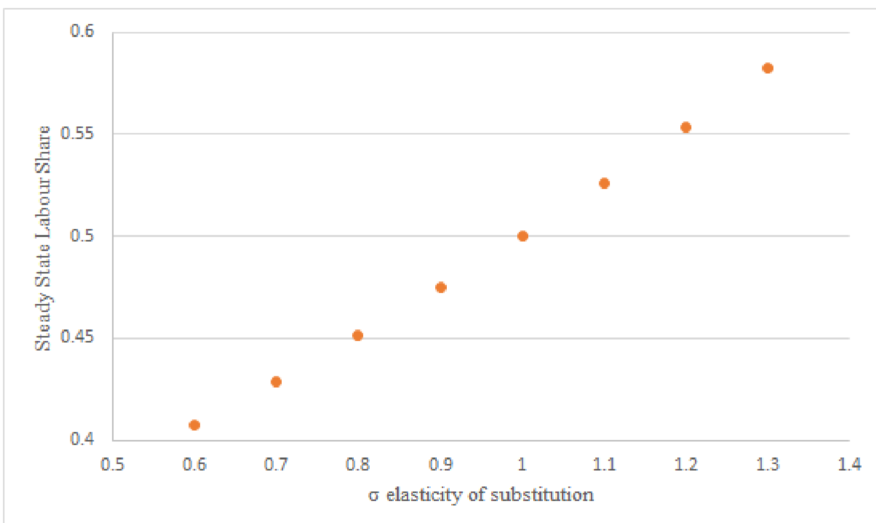
	$\sigma = 0.7$			$\sigma = 1.2$		
	$\gamma = 0.2$	$\gamma = 1.5$	$\gamma = 15$	$\gamma = 0.2$	$\gamma = 1.5$	$\gamma = 15$
LS	42.896	42.896	42.897	55.378	55.378	55.379
Y	11.66	0.19	0.16	61.05	8.14	0.81
I	1.44	0.19	0.02	18.65	0.50	0.05
L	16.67	2.22	0.22	112.69	15.03	1.50
S	7.22	0.96	0.10	18.65	2.49	0.25
K	10.26	1.37	0.14	49.05	6.54	0.65

Table 3.2: Steady state values for different levels of adjustment costs

Although the steady state level of the labor share is not sensitive to adjustment costs, the labor share remains not completely unaffected by it. Indeed, as we will see next, its response to price shocks varies across adjustment costs.

Another thing to note is that in our model the labor share is positively related with σ, as shown in Figure 3.1. If the factors are complements ($\sigma < 1$), then adjustment costs imply a labor share that is lower than in the Cobb Douglas case ($\sigma = 1$). When the factors are substitutes ($\sigma > 1$), the opposite is true. The intuition behind this finding is simple. Complementarity forces the firm to equalize the employment of the three factors while substitutability allows the firm to replace costly factors with cheaper ones. As the adjustment costs make intangible capital more costly than the other two factors, it is unsurprising that when factors are elastic the firm will employ the other two factors in greater proportions.

Figure 3.1: Steady State labor Share as a function of σ



3.3 Short-run dynamics and responses to price shocks

In order to analyze the interaction between adjustment costs and the labor share beyond the steady state; in this section, we consider the behavior of the different variables of our model in response to a price shock.

The impulse response functions (IRF) displayed in figure 3.2 show the deviations from the steady state in response to a unit price shock. The shape of the IRFs is independent of the substitutability of inputs. As a general rule, the price shock triggers a contraction in all real variables of the model. Investment falls contemporaneously with the price shock, and then gradually reverts back to its steady state level as the price shock dies out. Other variables (i.e. output (Y), tangible capital (K) intangible capital (S) and labor (L)) also go downwards immediately. However, unlike investment, they continue their fall in the subsequent periods, before eventually reverting back to their steady state values.

The dynamics causing the hump-shape and magnitude of the responses to the shock are quite intuitive, as they are a result of the persistence of the price shock and the accumulation of intangible capital. Due to our no time-to-build assumption, the unexpected rise in price of intangible capital leads to a decrease in investment at t=0. One can see from the FOCs 2.9 and 2.8 that labor and capital also drop at t=0. In next periods, the persistence of the price shock induces firms to continue investing less than in steady state. That is why all inputs continue decreasing with respect to their steady state levels for some periods more, before they start going back to their steady state levels. It is also important to notice the different levels of responsiveness in each variable. Labor is the input that drops the most while intangible capital is the "stickiest" input. The high responsiveness of labor is related to the factor shares assumed in our parametrization. Labor is the factor assigned the highest share ($a_L = 0.5$), while tangible and intangible capital are assigned a share of 0.25 each. Physical capital contracts more than intangible capital as it is rented every period rather than accumulated, and also not subject to adjustment costs.

Hence firms are able to adjust physical capital more flexibly when a price shock hits. To avoid the adjustment costs the firm prefers to let intangible capital depreciate rather then adjusting the stock downwards. When inputs are substitutes, the difference in the size of the responses is much more pronounced. Hence, firms prefer to adjust the other two inputs rather than disinvesting intangible capital in the short run and naturally do so more when inputs are more substitutable.

One important observation is that the response of the labor share is much smaller then that of real variables. Output and labor both display a contractionary response to the shock. As the labor share is a fraction of both they partially offset one another leading to a smaller response. In addition the response of the labor share is positive while all other variables respond negatively to the shock. Hence when investment becomes more costly, firms decrease their labor demand in absolute terms by less than the output drop. As explained above these dynamics do not translate to the long run.

3.3.1 The effect of changes in the adjustment costs

The responsiveness to price shocks is influenced by the level of adjustment cost. Figure 3.2a displays the variables response when inputs are substitutes. As the dynamics are not substantially different for complements we refer the reader to 5.2 Figure 5.1 for impulse response functions when inputs are complements.

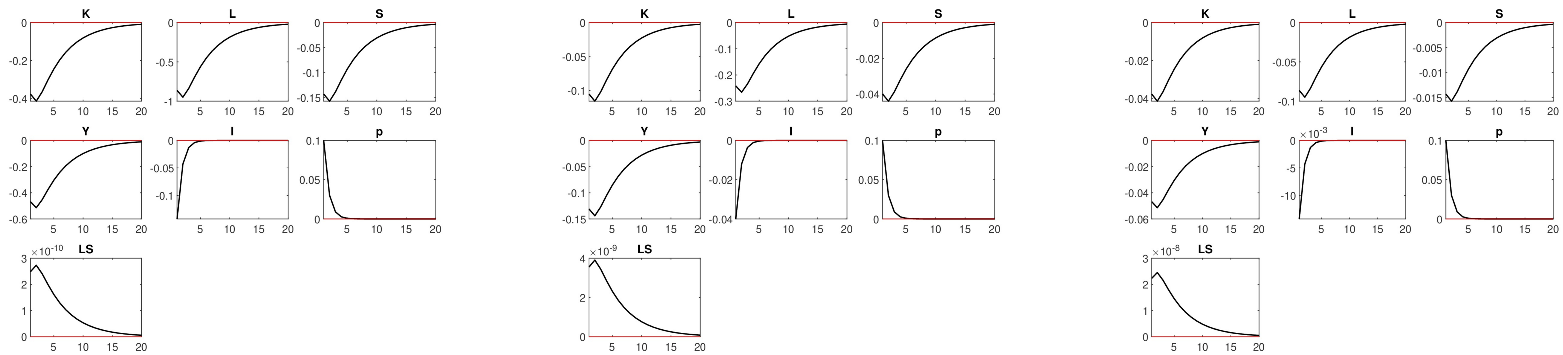
As one would expect the investment level becomes less responsive to price shocks as the cost of adjusting increases. These dynamics do not translate to the labor share: when inputs are substitutes as well as complements, the labour share becomes more responsive to price shocks as adjustment costs increase. As the adjustment costs increase, the smoothing incentive of firms regarding investment increases. While the responsiveness of all inputs to a price shock decreases with adjustment costs this effect is more pronounced for intangible capital. Hence labor becomes relatively more responsive than output. Thus, as adjustment costs become higher, the labor share spikes more in response to a price shock.

3.3.2 The role of volatility

Changing the level of volatility, that is, the variance of the process of p_t, does not influence any of the steady state values of our model. This is in contrast with the so called Oi-Hartman-Abel effect (Oi, 1961; Hartman, 1972; Abel, 1983) suggesting a higher steady state value of intangible capital in the presence of output price uncertainty. On the contrary, Pindyck suggests that with a constant second derivative of the adjustment cost function, as in our case, the steady state level of intangible capital will remain un-

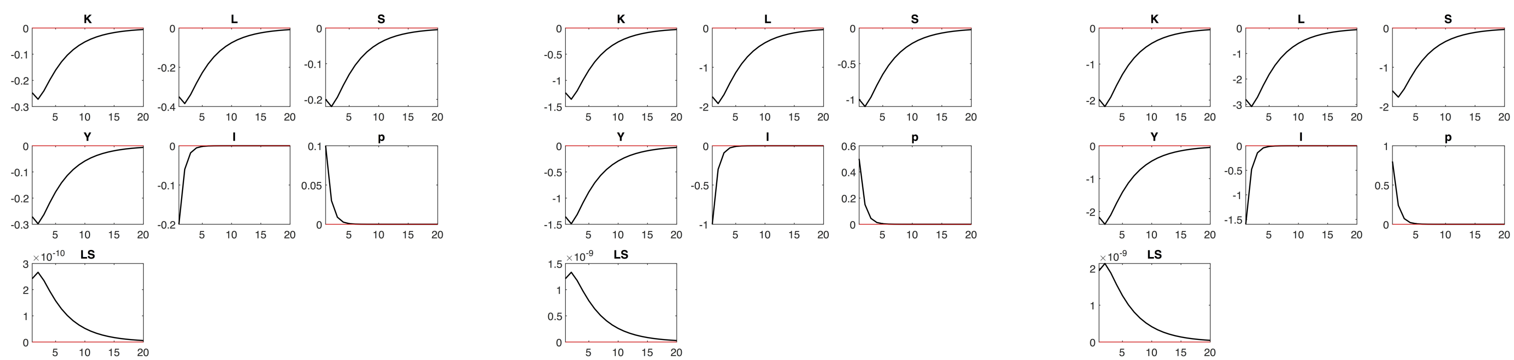
Figure 3.2: Impulse Response Functions

(a) Impulse Response Functions for different level of adjustment costs when inputs are substitutes ($\sigma = 1.2$)



$\gamma = 0.7$ $\gamma = 2.5$ $\gamma = 7$

(b) Impulse Response Functions for volatility levels of the stochastic process: 0.1, 0.5 and 0.8



Note: IRFS display deviations from steady state are in absolute terms. Panel (b): Elasticity of substitution, $\sigma = 0.5$, and adjustment costs parameter γ, of 0.5

affected when changing the volatility of the price process - precisely consistent with our findings for input price volatility (Pindyck, 1982).
It is important to highlight that volatility still plays a key role explaining the responsiveness of the different economic variables to a shock in prices.

Figure 3.2b is an illustration of how an increase in the variance of the process for p_t makes the labor share (LS) more responsive to a shock in price. Also a higher responsiveness is observed under other elasticities of substitution-including those above 1.
When variance of p_t rises, all responses are amplified: all variables drop by more as volatility increases, while the labor share increases more. Again, due to the interaction of the two "forces" driving the labor share movements (drop in output and in labor), the actual increase in labor share is very small.

The main insight from this section is, therefore, that even if adjustment costs might not play a role as drivers of the movements in the labor share; they could act as amplifiers of movements in the labor shares due to price shocks. It is interesting to link this conclusion with the literature suggesting that the decrease in price of capital has played a crucial role in driving the decrease of labor share (Karabarbounis and Neiman, 2013).

4 Conclusion

The answer to the question of whether adjustment costs of intangible capital can explain the decline in the labor share remains somewhat ambiguous. On the one hand, our simulations show that in our simple model there does not appear to be a meaningful relationship between the level of convex adjustment and the steady state labor share. Notably, this somewhat surprising result of our simulations can be intuitively explained in the context of equation 2.13 which links steady state level of intangible capital to the labor share. On the other hand, we have found that our model with adjustment costs leads to a very clear relationship between the elasticity of substitution and the labor share. Therefore, one could use our model to explain the secular decline in the labor share as a result of a falling elasticity of substitution in a presence of convex adjustment costs to intangible capital.

Moreover, we have found that adjustment costs affect a number of interesting short-run dynamics. We have demonstrated that the level of adjustment costs affects the responsiveness of the labor share to an exogenous shock to the price of intangible capital. In fact the level of adjustment costs affects the responsiveness of all three factors. And importantly, this effect depends on the elasticity of substitution in an intuitive way. Lastly we have shown that in our simple model the volatility of the price process does not alter the steady state labor share, even though it does matter for short run dynamics.

We see room for further research in the following directions. Our analysis assumes perfectly competitive markets. A model of monopolistic competition in the goods market could lead to long-run effects of the level of adjustment costs on the labor share. Karabarbounis and Neiman, 2013 showed that in such a model price decreases can explain part of the decrease in the labor share. Therefore, analysing the effect of adjustment costs in the context of monopolistic competition seems promising. Another potential avenue is the generalization of the analysis to a general equilibrium setting. Understanding endogenous changes in wages that were set to be fixed throughout our analysis, could be important in explaining the changes in the labor share.

5 Appendix

5.1 Derivations

The first order condition 2.10 with respect to intangible capital reads:

$$I_t = \frac{1}{\gamma}\left[a_S Y_t^{\frac{1}{\sigma}} S_t^{\frac{-1}{\sigma}} - p_t + \frac{1-\delta}{R} E_t\left[p_{t+1}\right]\right] + \frac{1-\delta}{R} E_t\left[I_{t+1}\right]$$

Rearranging the term, we obtain the following explicit equation for S_t and L_t

$$Y_t^{\frac{1}{\sigma}} S_t^{-\frac{1}{\sigma}} = \frac{I_t \gamma}{a_S} + \frac{1}{a_S}\left(p_t - \frac{1-\delta}{R} E_t[p_{t+1}]\right) - \frac{\gamma}{a_S}\frac{1-\delta}{R} E_t[I_{t+1}] \tag{5.1}$$

$$\left(a_K K_t^{\frac{\sigma-1}{\sigma}} + a_S S_t^{\frac{\sigma-1}{\sigma}} + a_L L_t^{\frac{1}{\sigma-1}}\right)^{\frac{1}{\sigma-1}} = S_t^{\frac{1}{\sigma}}\left[\frac{\gamma I_t + p_t - \frac{1-\delta}{R} E_t\left[p_{t+1}\right] - \gamma \frac{1-\delta}{R} E_t\left[I_{t+1}\right]}{a_S}\right] \tag{5.2}$$

$$\left(a_K K_t^{\frac{\sigma-1}{\sigma}} + a_S S_t^{\frac{\sigma-1}{\sigma}} + a_L L_t^{\frac{1}{\sigma-1}}\right) = S_t^{\frac{\sigma-1}{\sigma}} \Gamma_{t,t+1} \tag{5.3}$$

where

$$\Gamma_{t,t+1} = \left[\frac{\gamma I_t + p_t - \frac{1-\delta}{R} E_t\left[p_{t+1}\right] - \gamma \frac{1-\delta}{R} E_t\left[I_{t+1}\right]}{a_S}\right]^{\sigma-1} \tag{5.4}$$

which gives the following equations for intangible capital and labor:

$$S_t = \left(\frac{a_K K_t^{\frac{\sigma-1}{\sigma}} + a_L L_t^{\frac{\sigma-1}{\sigma}}}{\Gamma_{t,t+1} - a_S}\right)^{\frac{\sigma}{\sigma-1}} \tag{5.5}$$

$$L_t^{\frac{\sigma-1}{\sigma}} = \frac{S_t^{\frac{\sigma-1}{\sigma}}(\Gamma_{t,t+1} - a_S) - a_K K_t^{\frac{\sigma-1}{\sigma}}}{a_L} \tag{5.6}$$

Labor share

$$LS = \frac{w_t L_t}{Y_t} = a_L \left(\frac{Y_t}{L_t}\right)^{\frac{1-\sigma}{\sigma}} \tag{5.7}$$

Plugging in the equation for L_t:

$$LS = \frac{w_t L_t}{Y_t} = Y_t^{\frac{1-\sigma}{\sigma}}[S_t^{\frac{\sigma-1}{\sigma}}(\Gamma_{t,t+1} - a_S) - a_K K_t^{\frac{\sigma-1}{\sigma}}] \tag{5.8}$$

$$LS = \frac{S_t^{\frac{\sigma-1}{\sigma}}(\Gamma_{t,t+1} - a_S) - a_K K_t^{\frac{\sigma-1}{\sigma}}}{a_K K_t^{\frac{\sigma-1}{\sigma}} + a_S S_t^{\frac{\sigma-1}{\sigma}} + a_L L_t^{\frac{1}{\sigma-1}}} \tag{5.9}$$

Plugging in the equation for L_t again:

$$LS = \frac{S_t^{\frac{\sigma-1}{\sigma}}(\Gamma_{t,t+1} - a_S) - a_K K_t^{\frac{\sigma-1}{\sigma}}}{S_t^{\frac{\sigma-1}{\sigma}}\Gamma_{t,t+1}} = 1 - \frac{a_S}{\Gamma_{t,t+1}} - \frac{a_K K_t^{\frac{\sigma-1}{\sigma}}}{\Gamma_{t,t+1} S_t^{\frac{\sigma-1}{\sigma}}} \tag{5.10}$$

Next, consider FOC with respect to capital. This reads:

$$\frac{Y_t^{\frac{1}{\sigma}} a_K}{R} = K_t^{\frac{1}{\sigma}} \Rightarrow K_t^{\frac{\sigma-1}{\sigma}} = \frac{a_K^{\sigma-1} Y_t^{\frac{\sigma-1}{\sigma}}}{R^{\sigma-1}} \Rightarrow \frac{K_t^{\frac{\sigma-1}{\sigma}}}{S_t^{\frac{\sigma-1}{\sigma}}} = \frac{a_K^{\sigma-1}}{R^{\sigma-1}}\Gamma_{t,t+1} \tag{5.11}$$

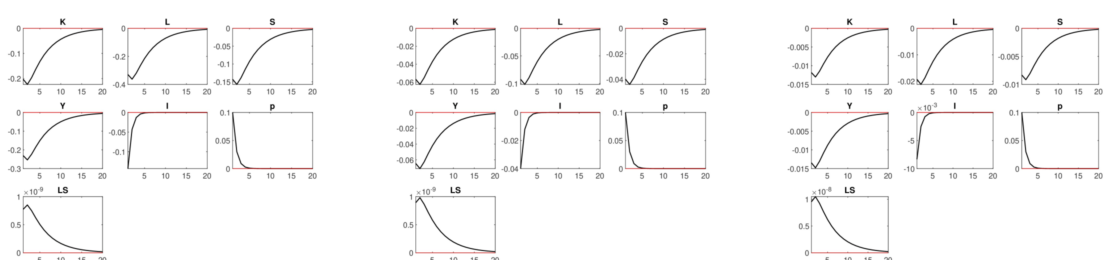
Hence the labor share equation becomes:

$$(LS)_t = 1 - \frac{a_S}{\Gamma_{t,t+1}} - \frac{a_K K_t^{\frac{\sigma-1}{\sigma}}}{\Gamma_{t,t+1} S_t^{\frac{\sigma-1}{\sigma}}} = 1 - \frac{a_S}{\Gamma_{t,t+1}} - \frac{a_K^{\sigma}}{R^{\sigma-1}} \tag{5.12}$$

$$(LS)_t = 1 - a_S^{\sigma}\left[\gamma I_t + p_t - \frac{1-\delta}{R}E_t\left[p_{t+1}\right] - \gamma\frac{1-\delta}{R}\left[I_{t+1}\right]\right]^{1-\sigma} - \frac{a_K^{\sigma}}{R^{\sigma-1}} \tag{5.13}$$

5.2 Additional IRFs

Figure 5.1: IRFs with different level of adjustment costs for complements ($\sigma = 0.7$)



$\gamma = 0.7$ $\gamma = 2.5$ $\gamma = 12$

References

Abel, Andrew B. (1983). 'Optimal Investment Under Uncertainty'. In: *The American Economic Review* 73.1, pp. 228–233. ISSN: 00028282. URL: http://www.jstor.org/stable/1803942 (cit. on p. 18).

Acemoglu, Daron and Pascual Restrepo (2017). 'Robots and Jobs: Evidence from US Labor Markets'. In: 23285. URL: https://www.nber.org/papers/w23285.pdf (cit. on p. 2).

Autor, David et al. (May 2017). 'Concentrating on the Fall of the Labor Share'. In: *American Economic Review* 107.5, pp. 180–85. DOI: 10.1257/aer.p20171102. URL: https://www.aeaweb.org/articles?id=10.1257/aer.p20171102 (cit. on p. 2).

Cette, Gilbert, Lorraine Koehl and Thomas Philippon (2019). 'The Labor Share in the Long Term: A Decline?' In: *Economie et Statistique / Economics and Statistics* 510-511-512, pp. 35–51. URL: https://www.insee.fr/en/statistiques/fichier/4253023/510_511_512_Cette_Koehl_Philippon_EN.pdf (cit. on p. 2).

Chiavari, Andrea and Sampreet Goraya (2020). 'The Rise of Intangible Capital and the Macroeconomic Implications'. In: *Working Paper, Department of Economics, Universitat Pompeu Fabra*. URL: shorturl.at/ENZ56 (cit. on p. 3).

Cooper, Russell W. and John C. Haltiwanger (July 2006). 'On the Nature of Capital Adjustment Costs'. In: *The Review of Economic Studies* 73.3, pp. 611–633. ISSN: 0034-6527. DOI: 10.1111/j.1467-937X.2006.00389.x. eprint: https://academic.oup.com/restud/article-pdf/73/3/611/18339620/73-3-611.pdf. URL: https://doi.org/10.1111/j.1467-937X.2006.00389.x (cit. on p. 12).

Growiec, Jakub, Peter McAdam and Jakub Mućk (2018). 'Endogenous labor share cycles: Theory and evidence'. In: *Journal of Economic Dynamics and Control* 87.C, pp. 74–93. DOI: 10.1016/j.jedc.2017.11.00. URL: https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp1765.en.pdf (cit. on p. 5).

Guerriero, Marta (2019). 'The Labor Share of Income around the World: Evidence from a Panel Dataset'. In: 920. URL: https://econpapers.repec.org/paper/risadbiwp/0920.htm (cit. on p. 4).

Hamilton, James D. (Dec. 2018). 'Why You Should Never Use the Hodrick-Prescott Filter'. In: *The Review of Economics and Statistics* 100.5, pp. 831–843. URL: http://www.mitpressjournals.org/doi/pdf/10.1162/rest_a_00706 (cit. on p. 4).

Hartman, Richard (1972). 'The effects of price and cost uncertainty on investment'. In: *Journal of Economic Theory* 5.2, pp. 258–266. ISSN: 0022-0531. DOI: https://doi.org/10.1016/0022-0531(72)90105-6. URL: http://www.sciencedirect.com/science/article/pii/0022053172901056 (cit. on p. 18).

Hayashi, Fumio (1982). 'Tobin's Marginal q and Average q: A Neoclassical Interpretation'. In: *Econometrica* 50.1, pp. 213–224. ISSN: 00129682, 14680262. URL: http://www.jstor.org/stable/1912538 (cit. on p. 12).

Hodrick, Robert J. and Edward C. Prescott (1997). 'Postwar U.S. Business Cycles: An Empirical Investigation'. In: *Journal of Money, Credit and Banking* 29.1, pp. 1–16. ISSN: 00222879, 15384616. URL: http://www.jstor.org/stable/2953682 (cit. on p. 4).

International Monetary Fund, . (Apr. 2017). 'World Economic Outlook: Gaining Momentum? - Chapter 3: Understanding the Downward Trend in Labor Income Shares'. In: *World Economic Outlook*, pp. 121–170. DOI: 10.5089/9781475597172.081. URL: https://www.imf.org/en/Publications/WEO/Issues/2017/04/04/world-economic-outlook-april-2017 (cit. on p. 2).

Kaldor, Nicholas (1955). 'Alternative Theories of Distribution'. In: *The Review of Economic Studies* 23.2, pp. 83–100. ISSN: 00346527, 1467937X. URL: http://www.jstor.org/stable/2296292 (cit. on p. 1).

Karabarbounis, Loukas and Brent Neiman (Oct. 2013). 'The Global Decline of the Labor Share*'. In: *The Quarterly Journal of Economics* 129.1, pp. 61–103. ISSN: 0033-5533. DOI: 10.1093/qje/qjt032. eprint: https://academic.oup.com/qje/article-pdf/129/1/61/30626867/qjt032.pdf. URL: https://doi.org/10.1093/qje/qjt032 (cit. on pp. 1, 2, 10, 12, 20, 21).

Koh, Dongya, Raül Santaeulàlia-Llopis and Yu Zheng (Sept. 2016). *Labor Share Decline and Intellectual Property Products Capital*. Working Papers 927. Barcelona Graduate School of Economics. URL: https://www.barcelonagse.eu/sites/default/files/working_paper_pdfs/927_1.pdf (cit. on p. 2).

McKinsey Global Institute, . (May 2019). 'A new look at the declining labor share of income in the United States'. In: *Discussion Paper*. URL: https://www.mckinsey.com/featured-insights/employment-and-growth/a-new-look-at-the-declining-labor-share-of-income-in-the-united-states (cit. on p. 2).

OECD (2018). *Decoupling of wages from productivity: what implications for public policies?* DOI: https://doi.org/https://doi.org/10.1787/eco_outlook-v2018-2-3-en. URL: https://www.oecd-ilibrary.org/content/component/eco_outlook-v2018-2-3-en (cit. on p. 1).

Oi, Walter Y. (1961). 'The Desirability of Price Instability Under Perfect Competition'. In: *Econometrica* 29.1, pp. 58–64. ISSN: 00129682, 14680262. URL: http://www.jstor.org/stable/1907687 (cit. on p. 18).

Pindyck, Robert S. (1982). 'Adjustment Costs, Uncertainty, and the Behavior of the Firm'. In: *The American Economic Review* 72.3, pp. 415–427. ISSN: 00028282. URL: http://www.jstor.org/stable/1831541 (cit. on p. 20).

Rognlie, Matthew (2015). 'Deciphering the Fall and Rise in the Net Capital Share: Accumulation or Scarcity?' In: *Brookings Papers on Economic Activity* 46.1 (Spring), pp. 1–69. URL: https://www.brookings.edu/wp-content/uploads/2016/07/1_2015a_rognlie.pdf (cit. on p. 2).

Sahin, Aysegül, Michael Elsby and Bart Hobijn (Sept. 2013). *The Decline of the U.S. Labor Share*. Working Paper Series 2013-27. Federal Reserve Bank of San Francisco. DOI: 10.24148/wp2013-27. URL: https://www.frbsf.org/economic-research/files/wp2013-27.pdf (cit. on p. 2).

Schneider, Dorothee (2011). 'The Labor Share: A Review of Theory and Evidence'. In: SFB649DP2011-069. URL: https://www.econstor.eu/bitstream/10419/56621/1/67118542X.pdf (cit. on p. 1).