MASTER PROJECT

On the effects of sovereign debt volatility: a theoretical model

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Abstract

We construct a theoretical Overlapping Generations (OLG) model to describe how sovereign debt crises can propagate in the economy under certain financial constraints. In the model, households work when young and deposit their savings in exchange for a dividend, banks invest deposits in assets and Government bonds. Banks, subject to legal and market requirements, invest a fixed fraction of deposits and own equity in assets. When prices of bonds fall due to perceived sovereign debt risks, banks can invest less on capital goods directly affecting the business cycle. This paper simulates the deviations from steady-state produced by a shock to government securities and provides insights into macro-prudential policy implications. We find that a sovereign debt crisis affects young and old generations differently, with the latter facing higher fluctuations in consumption. We also find that the macro-prudential policy can be effective only at very high levels on the old, but ineffective for the younger generation.

1 Introduction

Bubbles are an expression of the instability of financial markets and often precede economic crises. An expansionary bubbly episode is often characterized by a sudden change in the price of a given asset above its fundamental value, followed by a correction period when the price returns to its equilibrium value. Government securities can also be subject by volatile valuations due to market psychology. Especially in times of economic distress, governments are likely to issue more debt in order to finance budget deficits and fiscal stimuli. Monetary authorities help to allocate these bonds in the market and might plan to fill demand if the government's bonds undersell at a given market price. This is one of the tools available to central banks to stimulate the economy. Nonetheless, these market correction policies might fail to maintain the price of government securities at the desired value. Since these assets are traditionally deemed as safe, financial institutions are likely to hold these assets in relevant amounts. Thus, whenever there is a strong market correction on government bonds, banks are directly affected by it. More importantly, the distress of the financial system is quickly amplified and propagated to the real economy.

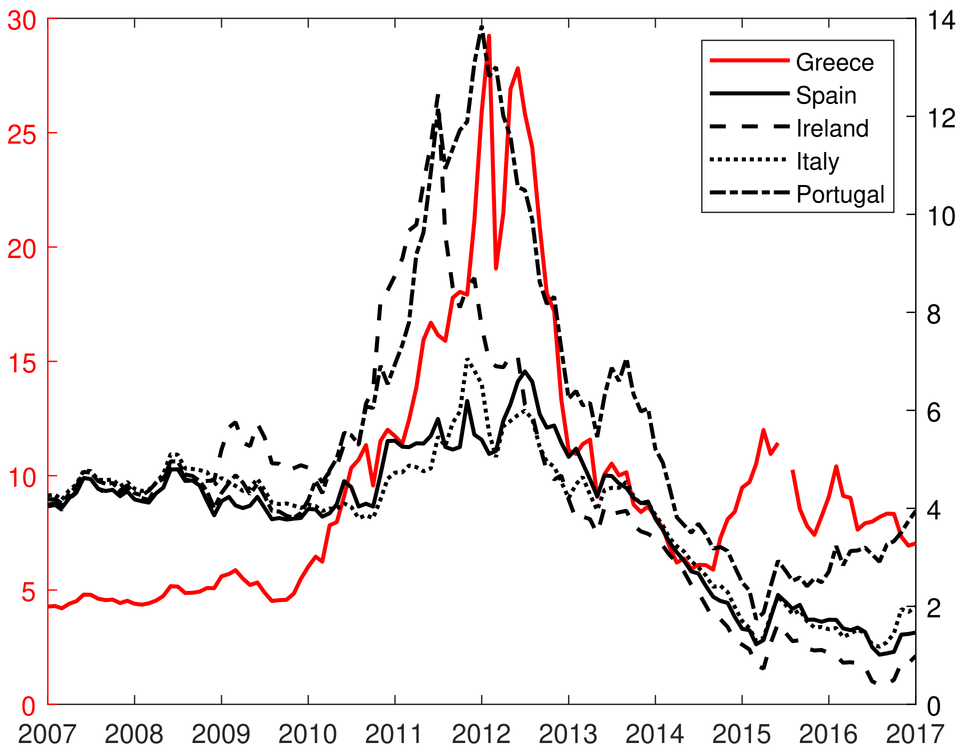


Figure 1: European Long-term Government Bond Yields (10-year)
Source: Elaboration on FRED *data*

A well-known example of these circumstances of financial distress is the sovereign debt crisis that has heavily impacted the countries of Southern Europe. From 2010, the government debt of Greece, Italy, Portugal, Ireland and Spain was under strict scrutiny of financial markets. The situation is shown clearly in Figure 1, where we plot the long-term government bonds yields for the aforementioned countries. It can be also noticed that, approximately, the period where these countries experienced the most volatility in the long term yield rates is between 2011 and 2013. Although the European Central Bank (ECB) attempted to intervene to some extent, these countries suffered economic downturns and fiscal restructuring. The mechanism through which the financial turmoil affected the dynamics of the business cycle is to be found in the rules imposed to financial institutions with regard to portfolio composition. In fact, banks must satisfy capital requirements proportional to the amount of risky assets and securities in their portfolio to avoid excessive leverage and over-exposure. However, these capital requirements do not insulate the real economy from the financial repercussions of sovereign debt crises. Especially in the presence of secular stagnation and limited effectiveness of monetary policy, policymakers have struggled to avoid and mitigate the consequences of financial distress on the real economy.

1.1 Research Question

This research project aims at analyzing how a sharp decline in government bond prices affects the business cycle through a simple overlapping generations model. The model contributes to the literature on financial institutions in its analysis of the financial constraints and mechanisms through which changes in asset prices propagate to the real economy. This paper analyzes the cyclical variations and co-movements of the main macroeconomic variables, how they are affected by abrupt changes in government bonds prices and what is the role of the financial constraints faced by banks in generating business cycles fluctuations. We want to investigate how the parametric assumptions we impose impact the evolution of consumption, investment and dividends. Furthermore, we aim to analyze the inter-generational distribution of welfare, how different generations, which we will call the "young" and the "old" for simplicity, are affected by the advent of a sovereign debt crisis. Ultimately, we would like our theoretical framework to provide insights on macro-prudential policy by simulating the path of a contractionary bubble in presence of different capital requirements for banks and how they impact the main macroeconomic variables.

1.2 Previous Literature

We aim to analyze the financial turmoil that has pervaded the sovereign debt markets in the last decade. Due to the contemporaneous great liquidity of financial markets and presence of informational asymmetry, the valuation of government bonds has seen great volatility in response to political and economic instability. In his seminal contribution, Tirole (1985), defines a bubble as the difference between the market value of an asset from its market fundamental; this oscillation is driven by the perception of increased risk and by market psychology. We focus our analysis on the volatility of government bonds, centering it on the interaction between banks and consumers. As for the financial system, the literature has focused on how the oscillations of the business cycle are amplified by the agency cost of lending as in Bernanke et al. (1999) and by the capital structure of the bank, as in Diamond and Rajan (2001).The institution of a common financial infrastructure could help improve international trust and stability, with positive impacts on welfare, as in Jeanne (2009). To understand the inter-generational and heterogeneous implications of bond prices on welfare, we adopt a similar framework as the one used by Martin and Ventura (2012) and by Galí (2021). But differently from the aforementioned, we will focus instead on the propagation of government debt volatility through the channel of bank balances. The comovement of the bond and credit market stems from the interrelation between sovereign and bank credit risk. Acharya et al. (2014) have analyzed this phenomenon, arguing that risk-taking behavior by the bank is induced by the possibility of government bail-out. On the other hand, excessive leverage would cause instability in the financial sector, which would pressure the government to intervene. Since expensive bailouts impact sovereign risk, they increase the fragility of the financial sector, now more susceptible to volatility. Additionally, government defaults do not bring only long-term damages, in terms of international incredibility, as shown by Acharya et al. (2014), but have pervasive negative impacts on the current economy through the balance sheets of domestic banks especially when institutions are integrated and developed (Gennaioli et al., 2018). To cite the case of Italy, which was severely impacted during the sovereign debt crisis, the spread between the yield rates of the Italian Bond and the German Bond had a pervasive impact on welfare, by reducing the supply of credit and the interest rate provided on deposits. During periods of financial distress, as informational asymmetry increases, purchases in domestic debt replace investments in the real economy, leading to a self-fulfilling crisis (Broner et al., 2014). Furthermore, Caprio and Honohan (1999) argue that capital standards are likely to be insufficient to assure banking stability for developing countries since banking authorities in emerging markets are thought to deal with larger real and financial disturbances. Regarding sovereign debts and capital requirements, Neyer and Sterzel (2017) study whether the introduction of capital requirements for bank government bond holdings increases financial stability by making the banking sector more resilient to sovereign debt crises. Making use of a theoretical model, they are able to show that rising capital requirements for government bonds actually increases the shock-absorbing capacity of the banking sector and thus the financial stability. However, the impact of asset price bubbles varies according to the dimension of banks and to their holder, specifically, they are amplified when it involves the banking system is damaged rather than ordinary savers (Aoki and Nikolov, 2015). Additional contributions to the literature on sovereign debt have analyzed the differential impact on debt maturity (Cole and Kehoe, 2000) and the impact of informational asymmetry on the incentive structure, as in Calomiris and Kahn (1991). Our model studies a small economy open to small international capital flows, which do not impact the financial dynamics of the rest of the world. Further research could analyze the implications of the volatile evaluations of government bonds on an international integrated market, such as Europe. Brutti and Sauré (2015) analyze precisely the effect of cross-border financial exposures of banks which have perverse effect in all the economies in an integrated international financial market. Clear enforcement of capital requirements and portfolio diversification could improve the efficacy of monetary policy. In this simplified setting, we analyze the impact of monetary policy in reducing the impact of a negative bubble on the business cycle, especially to the young generation. Similarly to Asriyan et al. (2021), which consider the interaction between bubbles and money, the welfare implications differ according to the source of the bubble.

1.3 Contribution and Rationale

The previous literature has focused on the mechanisms of propagation of a sovereign debt crisis and its effects on the real economy through heterogeneous agents models and in Dynamic Stochastic General Equilibrium (DSGE) models. We want to construct a theoretical framework to analyze the impact of a sovereign debt crisis on an economy where banks are the main agents financially constrained by capital requirements on portfolio composition.

The contribution of our research is to provide theoretical evidence of the impact of capital requirements on the propagation and amplification of sovereign debt crises. We contribute to the previous literature by constructing a simple OLG model that can analyze the behavior of financial markets along the business cycle, with special emphasis on the market instability of the past decade. Further, the rationale to undertake such research question is to investigate some possible macro-prudential policies and their heterogeneous effects on different generations in order to analyze who are most heavily impacted by such events and how authorities can intervene to offset the undesired consequences.

2 The Theoretical Model

The theoretical model we construct presents three main agents: households, firms and banks. Overall, they will respectively describe the consumption, production and financial sector sides of the economy. We then make a few assumptions on the nature of the shock and, finally, in the last section we model the exogenous asset prices process that produces our model's fluctuations. To get a better picture of the agents and actions they take in each period, we provide the following timeline of our model:

t					$t+\eta$	$t+2\eta$
Young work, save a_t, consume C_1^t	Old consume C_2^t	K_t formed from K_{t-1} and i_{t-1}	b_t realized	Banks pay d_t and operating costs, receive earnings, foreign investor cashes in e_t	Π_t^B turned into e_{t+1}, b_{t+1} bought by foreign investor	Banks invest i_{t+1}

Table 1: Timeline of the model

Let us now get into more details about what each component of our economy looks like.

2.1 Households

Consider a small economy inhabited by two overlapping generations, each of size L_t, which grow at a fixed exogenous rate n.

$$L_{t+1} = (1+n)L_t \tag{1}$$

Consumers live for two periods, where the young in the first period turn old in the second period. The young generation, the sole to be employed, inelastically supplies one unit of labor and is paid

w_t. The young save a fraction of their labor income in order to finance old-age consumption. Savings are deposited in the current account of the bank and yield a gross interest d_t. Hence, consumers only have to decide the optimal amount of savings a_t because investment decisions are carried out by the bank. Banks have perfect knowledge of asset and bond returns and locally benefit from increasing returns to scale, thus yielding more efficient investments. Lower-case variables are expressed in per-capita terms. Consumers' utility is logarithmic, such that:

$$\mathrm{U}(C_t) = \ln c_{1t} + \beta \ln c_{2t} \tag{2}$$

and subject to the following budget constraints of the form:

$$c_{1t} = w_t - a_t \tag{3}$$

$$c_{2t+1} = a_t d_{t+1} \tag{4}$$

Where β is the discount factor rate. By applying first order conditions, we end up having an expression for the optimal amount of savings:

$$a_t = \frac{\beta}{1+\beta} w_t \tag{5}$$

2.2 Production side of the economy

Consider for simplicity, a Neoclassical production function of the form: $F(K_t, L_t, \theta_t) = \theta_t K_t^{\alpha} L_t^{1-\alpha}$ where θ_t represents technology. Consequently, w_t (wages) and r_t (gross return of capital) are equal to the marginal product of labor and capital, respectively:

$$w_t = (1-\alpha)\theta_t K_t^{\alpha} L_t^{-\alpha} \tag{6}$$

$$r_t = \alpha \theta_t K_t^{\alpha-1} L_t^{1-\alpha} \tag{7}$$

Therefore, it is possible to combine the solution from the household's problem, where she maximizes savings and the firm's problem, in the following expression:

$$a_t = \frac{\beta}{1+\beta}(1-\alpha)\theta_t k_t^{\alpha} \tag{8}$$

Finally, the law of motion of capital is a product of the invested savings in productive assets i_t.

$$k_{t+1} = \frac{1}{1+n}(i_t a_t + k_t) \tag{9}$$

2.3 The Financial System

The productive sector comprises the financial system and firms. The foreign investor detains the ownership and control of the banks' stocks and of the physical capital formed. Each bank, unlike consumers, lives for an infinite period of time and operates in perfectly segmented markets. In each segment they act as monopolists due to proximity considerations and to the high fixed costs of switching. Banks make the investment decisions, choosing between the two types of assets. They either invest savings a_t in corporate assets i_t or in one-period government bonds b_t. The only source of volatility in the economy is given by the oscillations in the price of government bonds, although traditionally deemed as safe. The supervising institutions impose a liquidity requirement proportional to the amount of risky assets detained by the bank to ensure that the deposits of consumers are always risk-free. That is, banks can only invest up to a fraction x_t of its assets in producing capital goods, they must hold sufficient liquid assets (e_t and b_t) to avoid bank runs and mitigate the liquidity risk. The bank's equity e_t is equal to per-capita profit earned in t. We assume that a foreign investor buys the bonds and provides them to the bank. At the end of each period, it reinvests profits in the accumulated equity.

$$x_t \geq \frac{i_t}{e_t + b_t + i_t} \tag{10}$$

According to the capital requirements, we derive the minimum level of equity that must be held by the bank in order for her assets to be deemed safe.

$$e_t \geq \frac{1 - x_t}{x_t} i_t - b_t \equiv \underline{e} \tag{11}$$

By virtue of $x \in (0, 1)$, it holds that:

$$\frac{\partial \underline{e}}{\partial i_t} > 0 \tag{12}$$

$$\frac{\partial \underline{e}}{\partial b_t} < 0 \tag{13}$$

Therefore, an increase in investment in capital good increases the required amount of own stocks to be held by the bank, while an increase in the amount of bonds, by increasing the amount of safe assets held by the bank, softens the required amount of equity. For each period, t, banks maximize only $t + 1$ profits, as they are local monopolists on the funding market, but price takers on the integrated global financial market. Savers would optimally allocate their labor income to smooth consumption perfectly between their old and young age, independently of the interest rate. The fixed supply of savings, independent of the interest rate, could incentivize the bank to offer a negative interest rate, exploiting its monopoly power. Due to information asymmetries, banks do not know that the supply of savings is inelastic. Hence, they set d_{t+1} as if they were determining the a_t that maximizes profits. Banks also face operating costs, which are proportional to the dimensions of the bank, proxied by the amount of savings collected a_t. To take it into account, profits are reduced by subtracting earnings multiplied by γa_t, with $\gamma > 0$. This conveys the idea that increasing bank dimension (a_t) raises profitability up to the point where the marginal revenue is more than compensated by the augmenting operating costs. Hence, banks' profits can be expressed as:

$$\Pi^B_{t+1} = \left(\underbrace{\frac{(1+n)b_{t+1}}{b_t}}_{\text{bubble appreciation}} \underbrace{b_t L_t a_t}_{\text{investment in bond}} + r_{t+1} \underbrace{i_t L_t a_t}_{\text{investment in capital goods}} \right) - \underbrace{d_{t+1} a_t L_t}_{\text{dividend payout}} \tag{14}$$

$$\underbrace{-\gamma a_t \left(\frac{(1+n)b_{t+1}}{b_t} b_t L_t a_t + r_{t+1} i_t L_t a_t \right)}_{\text{operating costs}} \tag{15}$$

$$= \{[(1+n)b_{t+1} + r_{t+1} i_t](1 - \gamma a_t) - d_{t+1}\} \, a_t L_t \tag{16}$$

Putting together operating costs and earnings and defining the new term as rescaled earnings (see Appendix), the profit maximization problem can be visualized as in the following graph:

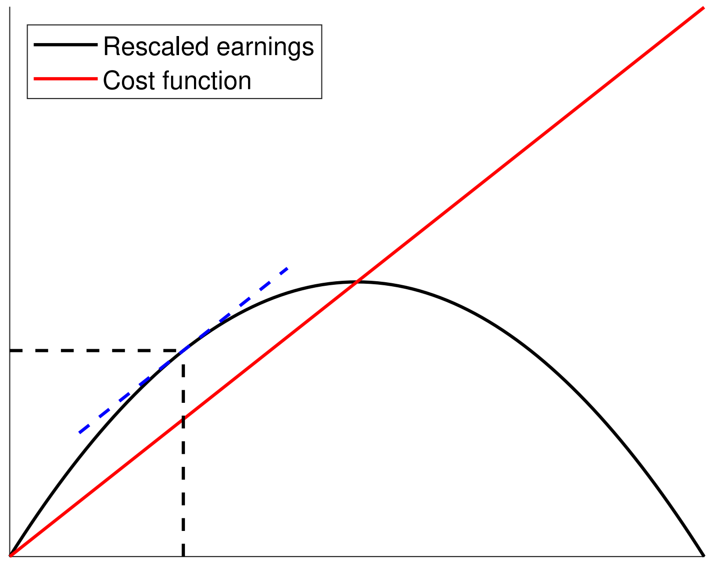


Figure 2: Earnings and cost curves for banks

Furthermore, both liquidity and real asset holdings e_{t+1} and i_{t+1} are positively affected by sovereign debt valuations, thus impacting the process of capital accumulation. Hence, rising (diminishing) bonds valuations positively (negatively) affect consumption both in the short run through the impact on interest payments and in the long run, through capital accumulation, which impacts the wages of the following generation.

2.4 Multiple equilibria

The investment decision made by the bank is affected by the presence of financial frictions and by the relative profitability of the two assets, independent of risk. Our standard arbitrage condition is expressed in per capita terms, thus involving (1+n) the growth rate of the bubble. It involves ϵ, which represents the moral suasion parameter from the central bank, which encourages investments in government bonds. We focus on the three equilibrium that might arise.

- If $\frac{(1+n)b_{t+1}}{b_t} > r_{t+1}+1+\varepsilon$, all resources are invested in government bonds. These unproductive investments cause the capital-labor ratio to decrease, until r_t surges to infinity, until r_t surges so much that the expression is satisfied with equality, unless b_t grows at a faster rate, in which case k_t will go to zero
- If $\frac{(1+n)b_{t+1}}{b_t} = r_{t+1} + 1 + \varepsilon$, there will be indifference between investing in the real economy and in government bonds, therefore, the equilibrium values cannot be determined.

- If $\frac{(1+n)b_{t+1}}{b_t} < r_{t+1} + 1 + \varepsilon$[1], our analysis will concentrate on this balanced equilibrium where both the real economy and bonds coexist. Investment in the real economy is more attractive, therefore, i_t will always be the highest possible. The lending constraint will hold with equality, as banks would invest in the real sector as much as allowed.

2.5 Government Bond Process

For the purpose of the computational simulations, we defined a bubble process in line with our research question which could replicate a sovereign debt crisis in our economy. Essentially, such a scenario can be reduced to a negative shock to government bonds prices. For simplicity, we decided to adopt a linear specification for the change in bond prices. We have attempted to recreate the contraction experienced by the prices of sovereign debt in the Eurozone in the early 2010. The bubble process declines for 50 time periods, then stays flat for an analogous amount of time, and then we simulate a "correction period" where it returns to its initial value, from period 100 onward. The movement can be visualized in the following figure:

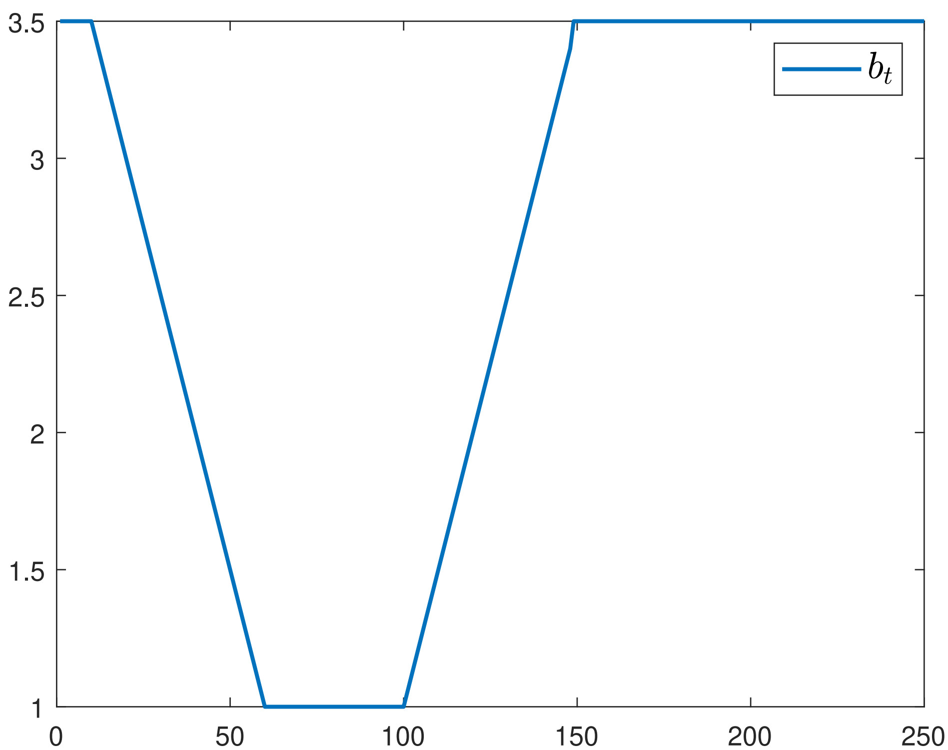


Figure 3: Negative Shock to Bonds for Simulations.

[1]The expression derives from:

$$\frac{B_{t+1}}{B_t} = \frac{B_{t+1}/L_t}{B_t/L_t} = \frac{(1+n)B_{t+1}/[(1+n)L_t]}{b_t} = \frac{(1+n)b_{t+1}}{b_t}$$

Instead, in Tirole (1985) the no-arbitrage condition states $\frac{(1+n)b_{t+1}}{b_t} = 1 + r_{t+1}$, which must hold, otherwise risk-neutral agents would not hold the bubble. However, here banks are given the bond by the foreign investor (and hence consider its cost as null) and are obliged to hold it if they want to invest more than allowed by e_t.

It is important to consider that by period 150 the value of government bonds returns to the value of 3.5 and the negative bubble is extinguished. The bubble process which impacts the decrease in bond prices is treated as exogenous, and its value is completely arbitrary. However, we made sure that the bond prices do not go below zero, which would not be allowed by the supervising financial institutions. For all the simulations that will follow, unless differently specified, we will employ this specific negative bubble process as the shock to the economy in our model.

3 Results

We simulate the model in MATLAB with a standard calibration of the parameters (see Table 2), and we predetermine capital k_0 for each of the following simulations. From our theoretical framework, we focus on the scenario where the government bond value growth rate is low enough to avoid the degenerate equilibrium where all savings are invested in government bonds, reaching a limit case where its aggregate value overgrows the economy. In order to check how the growth rate of the government bond value evolves compared to the rate of returns for capital goods, we compute the following graph representing the evolution of the two conditions:

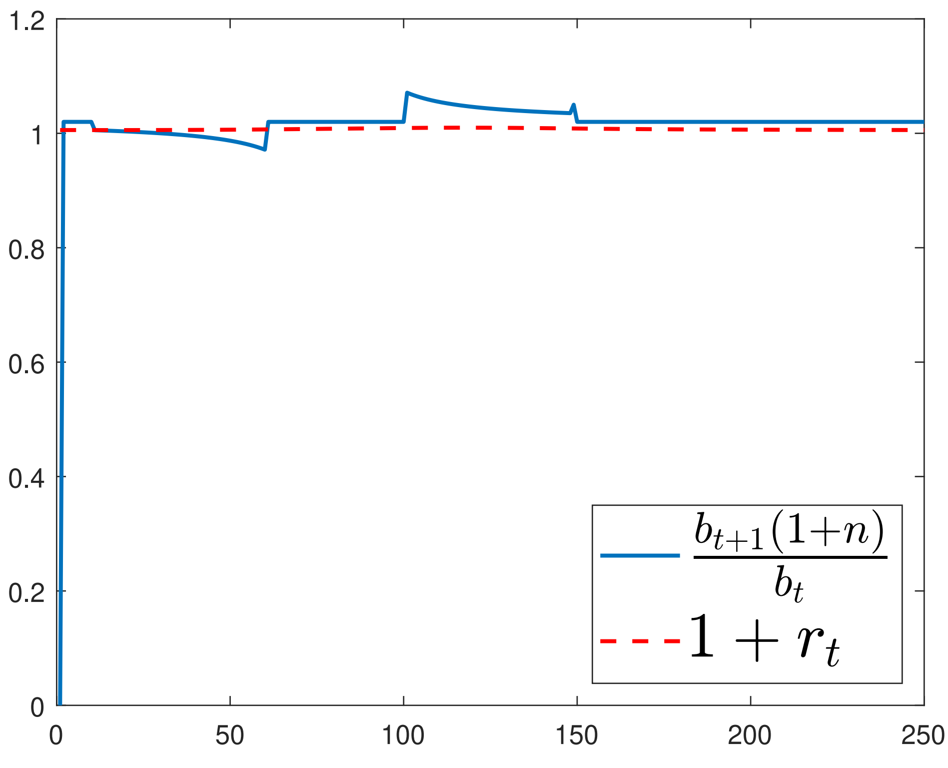


Figure 4: Local Indifference Condition (for $x = 0.5$).

Even for small ε and several values for capital requirements x (Figure 12 and 13), the assumption that holding the capital good is profitable appears to be realistic for the whole lifetime of the economy.

Accordingly, we will analyze the simulation results for different values of the capital requirements x, but we will not consider any case below 0.5, to maintain adherence to reality. For example, in the Eurozone, the Basel requirements on equity are lower than 10%, so banks hold relatively a small fraction of government bonds. For this reason, our analysis will focus on values upwards of 0.5 and as high as 0.9. The negative shock to government bonds prices b_t leads to the following path of the model's main macroeconomic variables:

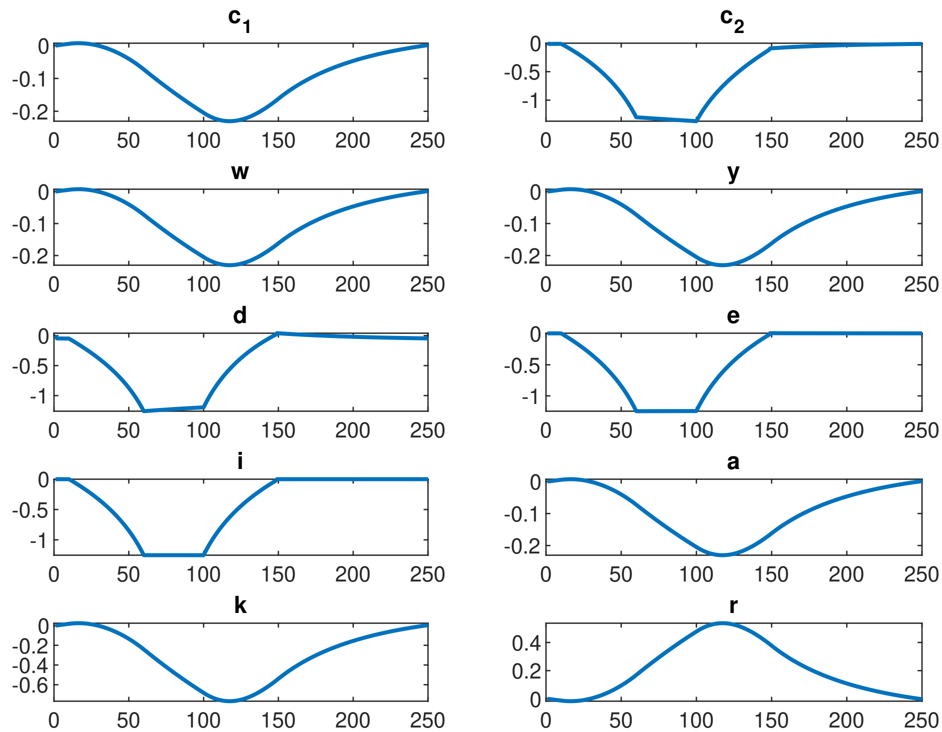


Figure 5: Propagation for $x = 0.5$ (Log-Deviations from SS).

Figure 5 shows a simulation of the propagation of a sovereign debt crisis in the economy. The simulation shows that all the variables, apart from returns to capital, are procyclical with respect to per capita bond prices b_t. As soon as a negative shock in government bond prices protracts, present and future consumption begin to decrease. They do so differently: present consumption seems to follow a smoother path, whereas future consumption bottoms and recovers much faster. Additionally, the fall in bond prices lead to lower investment in capital goods which, in turn, causes the household to start depleting the capital stock. The decreasing capital stock determines the path for the only countercyclical variable in our theoretical model - the returns to capital - since the marginal product of capital rises during the propagation of the government bonds crisis. Once b_t begins to rise again, capital stock bounces back and the returns to capital begin to rise again. As we will see later, not all variables behave equally when considering their volatility. In fact, the highlighted macroeconomic variable in Figure 5 shows the log-deviations from their steady-state values. For the sake of completeness, we also produce the path of variables in the case of a positive shock to government bonds prices in Figure 6.

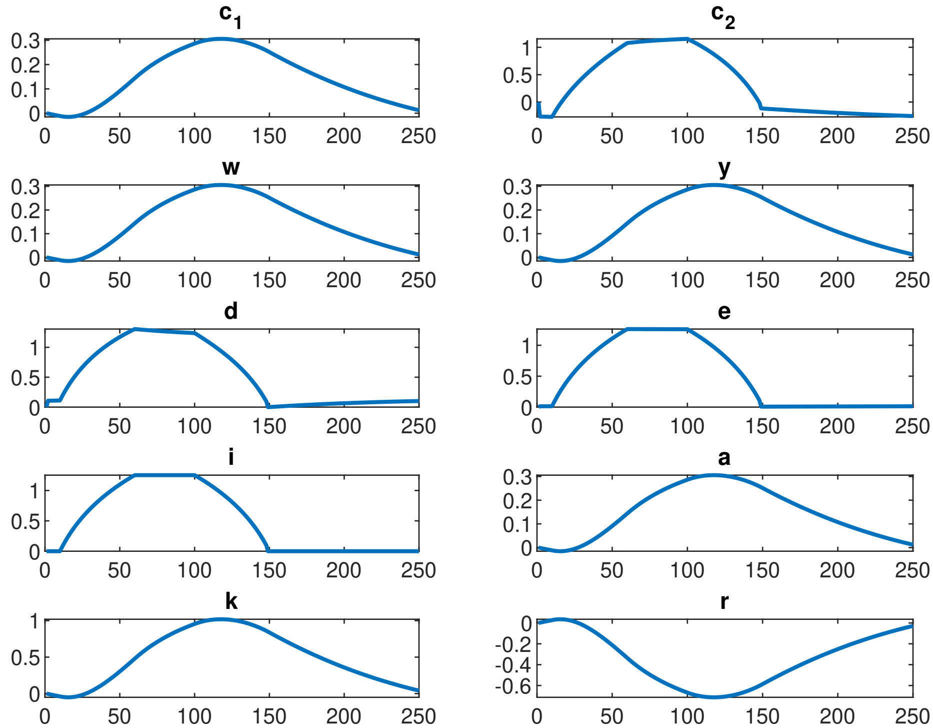
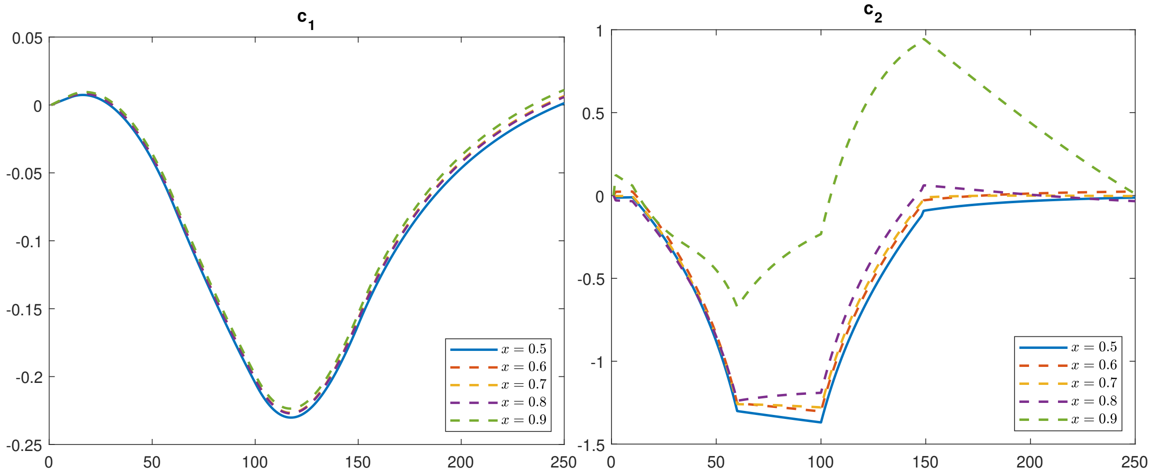


Figure 6: Propagation for $x = 0.5$ (Log-Deviations from SS, Opposite Bond Path).

As aforementioned, in our theoretical economy, the effect of a sovereign debt crisis on present and future consumption differs. In particular, if we look at their paths, the former follows a smoother curve which reaches a minimum at around period 120 to then return to its steady-state value after 250 periods. Conversely, the latter shows a very different transition that bottoms much earlier (at $t = 60$) but recovers much earlier too. There are also important differences in the size of the log-deviations from steady-state levels. In fact, future consumption seems to deviate more than present consumption. The difference between the two is to be found in the path of dividends, which resembles exactly the one for c_2, whilst c_1 follows the path of wages and assets. This is the case because consumption of the young depends on their wages and the assets they purchase, whereas consumption of the old relies upon future dividends and future assets. When a sovereign debt crisis takes place, production and wages decrease slower but also take longer to return to their steady-state values. Conversely, equity and dividends change much more abruptly but also go back to steady-state much earlier. This carries important implications for the effect of a sovereign debt crisis on the consumption of young and old generations. Young generations face a much smoother path away and back to steady-state, whereas old generations face much more sudden and quick log-deviations but a faster recovery.

This situation can be visualized in Figure 7 where the paths of c_1 and c_2 are plotted for different values of capital requirements x. These graphs for the remaining variables can be found in the section Appendix (Figures 8-11). Here we focus on the development of consumption:



(a) Consumption of the Young

(b) Consumption for the Old

Figure 7: Consumption for Young and Old
(Log-Deviations from SS for Different Levels of x).

The results show a clear picture of how our suggested macro-prudential policy impacts the business cycle: a change in capital requirements for banks, affects consumption for young and old generations. In the case of c_2, changing the requirements banks have to follow have a little impact for x between 0.5 and 0.8. Within this range, there is little impact on the log-deviations of consumption of the old from its steady-state values. However, this is different for a policy of $x = 0.9$ where the share of banks' portfolios that can be invested in capital goods is very high. This scenario generates higher fluctuations in dividends, thus impacting changes in future consumption, c_2. It is not clear whether this level of x is more desirable than other policies, it can only be observed that the path of consumption for the old under this macro-prudential policy is very much different from at other levels. Old generations would face higher fluctuations, although there is a reduction in the negative log-deviations from steady-state due to the slump in government bonds prices. On the other hand, consumption for the young c_1 follows a much smoother path, depending on the evolution of wages and output as shown in Figure 8. These two variables are not heavily impacted by changes in the capital requirements of the bank and, therefore, when per-capita bond prices b_t decrease present consumption remains similar for all values of the macro-prudential parameter x. This implies that consumption for the young generation is unaffected by any policy the supervising authority might want to pursue. Either way, the impact of a sovereign debt crisis on the young will remain consistent. Unlike c_1, the evolution of c_2 will be greatly affected for very high values of x, which gives some room for macro-prudential policy.

Finally, our theoretical model provides some insights about the limit case in which the value of government securities goes to zero ($b_{ss} = 0$), which is worth mentioning. In this case, the level of steady-state capital is:

$$k_{ss} = \left[\frac{\gamma \alpha x \theta^3}{(1+n)(1-x)} \left(\frac{\beta(1-\alpha)}{1+\beta} \right)^2 \right]^{\frac{1}{1-3\alpha}} \tag{17}$$

The value of the capital-labor ratio should be calculated by substituting in for the values of parameters. However, since $0 < \beta < 1$ and assuming $\alpha = \frac{1}{3}$ as widely established in the literature, $k_{ss} \to 0$ for a wide range of parameters. In particular, this is always the case when $\frac{\gamma \alpha x \theta^3}{(1+n)(1-x)} < 1$. The result is consistent with the fact that the lending constraint becomes so binding that investment in the real economy is so low, resulting in the population outgrowing capital. In other words, when the value of government bonds approaches 0, the capital requirements become increasingly stricter for banks. These results, unless under unrealistic and extreme parametric assumptions, in no per-capita capital in the steady-state.

Overall, these results can have important policy implications for how the supervising authority might want to react to the sovereign debt crisis, depending on the objectives it wants to achieve. Institutions that want to smooth the downturns in the consumption of the old might not be affecting the impact of the crisis on the young. In our framework, although there is no welfare loss function for the supervisory authority, it appears to be always beneficial to pursue an aggressive policy on capital requirements when the economy is facing a sovereign debt crisis. However, note that our theoretical model does not assume price rigidities so that they can perfectly adjust. Furthermore, note that all the previous considerations on the actions available to the supervising authority hold true as long as she imposes sufficiently prudent criteria in order to avoid bank failures and runs. Otherwise, there are further modelling conditions that we need to include and satisfy.

4 Conclusions

4.1 Findings

This paper draws three main conclusions about the impact of a sovereign debt crisis on the business cycle within the proposed OLG theoretical framework.

First, a decline in government bond prices leads to lower output, wages and dividend negatively affecting c_1 and c_2. However, this effect is different for young and old generations. In particular, the old seem to face more sudden changes and higher deviations from steady-state values when a sovereign debt crisis takes place. They are subject to bigger declines within few time periods of the economy's lifetime. However, consumption for the old also recovers much faster than that of the young.

Second, the proposed macro-prudential policy does not seem to offset the impact of a fall in government bonds prices on the business cycle. In fact, almost all the macroeconomic variables of interest in our theoretical model do not change significantly, relative to their steady-state values, when the supervising authority modifies the capital requirements for banks. Simulation results show log-deviations from steady-state for output, wages and consumption of the young remain almost unchanged for different intensity of the macro-prudential policy.

Third, a very aggressive policy on capital requirements (i.e. $x = 0.9$ for the whole period) can compensate for the negative shock bonds prices have on dividends and, therefore, consumption for the old. As shown in Figure 7, the strong impact that an aggressive macro-prudential policy has on dividends pushes c_2 above steady-state value during part of the sovereign debt crisis. This comes after a first decline in consumption as lower bond prices hit the economy.

4.2 Implication and Further Research Questions

The main implications of our research paper derive from the mechanism through which a sovereign debt crisis propagates in the real economy, affecting the business cycle. We have shown that in the presence of a sudden decrease in per capita bonds price b_t, young generations fare better in terms of deviation from their steady-state levels, whereas old generations experience higher volatility and abrupt changes to their consumption. However, our results also suggest that the supervising authority can increase the future generation's welfare through the increment of the share of assets banks can invest in capital goods. Very aggressive policies can offset the impact of the sovereign debt crisis on future dividends, helping future consumption to recover. In addition, the findings imply that the same policy is not effective in compensating the downturn in output and wages and, therefore, failing to close the negative fluctuations in present consumption.

The presented theoretical model has many more avenues to discover for further research purposes. For example, there is much room for parametric calibration of the model. Perhaps, adjusting some parameters in the model can produce fluctuations that resemble those of real-life scenarios and this can help the analysis of the business cycle and policy in the presence of a sovereign debt crisis. Further, there is room for an in-depth analysis with the introduction of technological shocks θ in our production function. Another possibility of extending this framework could be the introduction of a central bank that follows a policy rule and the inclusion of a public sector. These two entities would provide useful tools to analyze further policies that could help to smooth the effects of a crisis on sovereign debt, depending on the goals the institution cares to achieve. Moreover, a shortcoming of our model is the absence of price rigidities that could create frictions in the labor market and on the production side of the economy. In more details, given that our case studies the fall in government bonds prices and wages have to fall in order for markets to clear, the presence of price rigidity would create involuntary unemployment if wages cannot fall enough. This further research prospect could also provide useful policy tools, such as government transfers, taxes and monetary policy, to counter the effects of decreasing government bonds prices on output and wages in order to improve the welfare of the young. Finally, this theoretical framework could be applied for other types of financial crises and propagation scenarios by adjusting the nature of the bubble. We have described this process as a linear decline in government bonds prices followed by an increase, but this could be adapted to accommodate several scenarios.

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5 Appendix

5.1 The model

The workers' utility maximization problem looks as follows:

$$\max_{c_{1t},c_{2t+1}} \quad \ln c_{1t} + \beta\mathbb{E}\left(\ln c_{2t+1}\right)$$

$$\text{s.t.} \quad c_{1t} = w_t - a_t \tag{18}$$

$$c_{2t+1} = d_{t+1}a_t$$

$$\max_{a_t} \quad \ln(w_t - a_t) + \beta\ln(d_{t+1}a_t) \tag{19}$$

$$\text{FOC:} \quad \frac{\partial u_t}{\partial a_t} = -\frac{1}{w_t - a_t} + \beta\mathbb{E}\left(\frac{d_{t+1}}{d_{t+1}a_t}\right) = 0 \tag{20}$$

$$\frac{1}{w_t - a_t} = \frac{\beta}{a_t} \tag{21}$$

$$a_t = \frac{\beta}{1+\beta}w_t \tag{22}$$

Normalizing the price of the final good to one (numéraire) and assuming there is no capital depreciation:

$$\max_{K_t,L_t} \quad \theta_t K_t^{\alpha} L_t^{1-\alpha} - r_t K_t - w_t L_t \tag{23}$$

$$\frac{\partial \Pi_t}{\partial K_t}: \quad \alpha\theta_t K_t^{\alpha-1} L_t^{1-\alpha} - r_{=}0 \tag{24}$$

$$r_t = \alpha\theta_t k_t^{\alpha-1} \tag{25}$$

$$\frac{\partial \Pi_t}{\partial L_t}: \quad (1-\alpha)\theta_t K_t^{\alpha} L_t^{-\alpha} - w_t = 0 \tag{26}$$

$$w_t = (1-\alpha)\theta_t k_t^{\alpha} \tag{27}$$

Hence:

$$a_t = \frac{\beta}{1+\beta}(1-\alpha)\theta_t k_t^{\alpha} \tag{28}$$

Financial institutions can only invest a fraction of the funds collected in the real sector due to legal or market requirements. Namely, in order for bank bonds to be deemed "safe" (whether by supervising institutions or by the market), the bank can only invest up to x_t of its assets in producing capital goods, where:

$$x_t = \frac{i_t}{e_t + b_t + i_t} = \frac{i_t}{e_t + a_t} \tag{29}$$

$$xi_t + xb_t + xe_t - i_t = 0 \tag{30}$$

$$(x-1)i_t = -x(b_t + e_t) \tag{31}$$

$$i_t = \frac{x(b_t + e_t)}{1 - x} \tag{32}$$

Note that a_t, b_t, i_t, e_t are in per capita terms. Here, i_t is capital investment, producing $i_t a_t L_t$ units of capital in $t+1$ and e_t is the equity of the bank, equal to per-capita profits in t. b_t is the value of the bubble at time t.

Hence, their profits would be:

$$\Pi^B_{t+1} = e_{t+1} L_{t+1} \tag{33}$$

$$= \left(\underbrace{\frac{(1+n)b_{t+1}}{b_t}}_{\text{bubble appreciation}} \underbrace{b_t L_t a_t}_{\text{investment in bond}} + r_{t+1} \underbrace{i_t L_t a_t}_{\text{investment in capital goods}} \right) - \underbrace{d_{t+1} a_t L_t}_{\text{dividend payout}} \tag{34}$$

$$\underbrace{-\gamma a_t \left(\frac{(1+n)b_{t+1}}{b_t} b_t L_t a_t + r_{t+1} i_t L_t a_t \right)}_{\text{operating costs}} \tag{35}$$

$$= \underbrace{[(1+n)b_{t+1} a_t L_t + r_{t+1} i_t a_t L_t](1 - \gamma a_t)}_{\text{rescaled earnings}} - d_{t+1} a_t L_t \tag{36}$$

$$= \{[(1+n)b_{t+1} + r_{t+1} i_t](1 - \gamma a_t) - d_{t+1}\} a_t L_t \tag{37}$$

$$e_{t+1} = \{[(1+n)b_{t+1} + r_{t+1} i_t](1 - \gamma a_t) - d_{t+1}\} \frac{a_t L_t}{L_{t+1}} \tag{38}$$

$$= \{[(1+n)b_{t+1} + r_{t+1} i_t](1 - \gamma a_t) - d_{t+1}\} \frac{a_t}{1+n} \tag{39}$$

At time t, banks maximize $t+1$ profits with respect to savings collection in t. The optimal condition entails that dividend payments (d_t) are:

$$\frac{\partial \Pi^B_{t+1}}{\partial a_t} = \{[(1+n)b_{t+1} + r_{t+1} i_t](1 - 2\gamma a_t) - d_{t+1}\} L_t = 0 \tag{40}$$

Which implies that:

$$d_{t+1} = [(1+n)b_{t+1} + r_{t+1} i_t](1 - 2\gamma a_t) \tag{41}$$

Thus, for higher bonds values at $t+1$, c_2 augments at $t+1$ ceteris paribus. Conversely, for lower b_{t+1}, c_{2t+1} diminishes. Substituting in for d_{t+1} in the expression for e_{t+1}:

$$e_{t+1} = \frac{a_t}{1+n} \{[(1+n)b_{t+1} + r_{t+1} i_t](1 - \gamma a_t) - [(1+n)b_{t+1} - b_t + r_{t+1} i_t](1 - 2\gamma a_t)\} \tag{42}$$

Then:

$$e_{t+1} = \frac{\gamma a_t^2}{1+n} [(1+n)b_{t+1} + r_{t+1} i_t] \tag{43}$$

Since e_{t+1} is increasing in the value of the bonds at $t+1$, also i_{t+1} is, affecting k_{t+2}. Hence, rising (diminishing) bubble valuations positively (negatively) affects consumption both in the short and medium run through the impact on interest payments and on capital accumulation, and hence wages.

Steady State Equations

Now, look at the steady-state equilibrium, starting from the expressions for capital-labor ratio and substituting in for the value of a_t.

$$\begin{aligned} k_{t+1} &= \frac{1}{1+n}\left(i_t a_t + k_t\right) \\ &= \frac{1}{1+n}\left(i_t \frac{\beta}{1+\beta}(1-\alpha)\theta_t k_t^{\alpha} + k_t\right) \end{aligned} \tag{44}$$

Assume that in the steady state productivity and balance sheet requirements are fixed such that $\theta_t = \theta$ and $x_t = x$, as well as $k_t = k$ and $i_t = i$. It then follows that:

$$k_{ss}^{1-\alpha} = \frac{\beta(1-\alpha)\theta i_{ss}}{n(1+\beta)} \tag{45}$$

$$k_{ss} = \left(\frac{\beta(1-\alpha)\theta i_{ss}}{n(1+\beta)}\right)^{\frac{1}{1-\alpha}} \tag{46}$$

Solving for the steady state of the other variables:

$$r_{ss} = \alpha\theta\left(\frac{\beta(1-\alpha)\theta i_{ss}}{n(1+\beta)}\right)^{\frac{\alpha-1}{1-\alpha}} \tag{47}$$

$$= \alpha\theta\left(\frac{\beta(1-\alpha)\theta i_{ss}}{n(1+\beta)}\right)^{-1} \tag{48}$$

$$= \frac{\alpha n(1+\beta)}{\beta(1-\alpha)i_{ss}} \tag{49}$$

$$w_{ss} = (1-\alpha)\theta\left(\frac{\beta(1-\alpha)\theta i_{ss}}{n(1+\beta)}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{\beta(1-\alpha)^{\frac{1}{\alpha}}\theta^{\frac{1}{\alpha}} i_{ss}}{n(1+\beta)}\right)^{\frac{\alpha}{1-\alpha}} \tag{50}$$

$$a_{ss} = \frac{\beta}{1+\beta}\left(\frac{\beta(1-\alpha)^{\frac{1}{\alpha}}\theta^{\frac{1}{\alpha}} i_{ss}}{n(1+\beta)}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{\beta(1-\alpha)\theta i_{ss}^{\alpha}}{n^{\alpha}(1+\beta)}\right)^{\frac{1}{1-\alpha}} \tag{51}$$

In order to find an expression for e_{ss}, introduce ω such that $\frac{1+n}{\omega-1}b_{ss} = r_{ss}i_{ss}$. Hence:

$$b_{ss} = \frac{\alpha n(\omega-1)(1+\beta)}{\beta(1+n)(1-\alpha)i_{ss}} i_{ss} \tag{52}$$

$$e_{ss} = \frac{\gamma}{1+n}\frac{\beta^2}{(1+\beta)^2}(1-\alpha)^2\theta^2\left(\frac{\beta(1-\alpha)\theta i_{ss}}{n(1+\beta)}\right)^{\frac{2\alpha}{1-\alpha}}\left[(\omega-1)\frac{\alpha n(1+\beta)}{\beta(1-\alpha)(1+n)} + \frac{\alpha n(1+\beta)}{\beta(1-\alpha)(1+n)}\right] \tag{53}$$

$$= \frac{\gamma}{1+n}\left(\frac{i_{ss}}{n}\right)^{\frac{2\alpha}{1-\alpha}}\left(\frac{\beta\theta(1-\alpha)}{1+\beta}\right)^{\frac{2}{1-\alpha}}\left[\omega\frac{\alpha n(1+\beta)}{\beta(1-\alpha)(1+n)}\right] \tag{54}$$

$$= \frac{\gamma}{1+n}\left(\frac{i_{ss}}{n}\right)^{\frac{2\alpha}{1-\alpha}}\left(\frac{\beta\theta(1-\alpha)}{1+\beta}\right)^{\frac{2}{1-\alpha}}\left[\omega\frac{\alpha n(1+\beta)}{\beta(1-\alpha)(1+n)}\right] \tag{55}$$

$$= \frac{\omega\gamma\alpha}{(1+n)^2}\left[\theta^2 n^{1-3\alpha}\left(\frac{\beta(1-\alpha)}{1+\beta}\right)^{1+\alpha} i_{ss}^{2\alpha}\right]^{\frac{1}{1-\alpha}} \tag{56}$$

Setting $\alpha = \frac{1}{3}$:

$$
\begin{aligned}
e_{ss} &= \frac{\omega\gamma}{3(1+n)^2}\left[\theta^2\left(\frac{2\beta}{3(1+\beta)}\right)^{\frac{4}{3}} i_{ss}^{\frac{2}{3}}\right]^{\frac{3}{2}} && (57)\\
&= \frac{\omega\gamma}{3(1+n)^2}\theta^3\frac{4\beta^2}{9(1+\beta)^2}i_{ss} && (58)\\
&= \frac{4\omega}{27}\frac{\gamma\beta^2\theta^3}{(1+n)^2(1+\beta)^2}i_{ss} && (59)\\
& && (60)
\end{aligned}
$$

Noticing that:

$$
\begin{aligned}
i_{ss} &= \frac{x}{1-x}\left(\frac{(\omega-1)\alpha n(1+\beta)}{\beta(1-\alpha)(1+n)} + e_{ss}\right) && (61)\\
&= \frac{x}{1-x}\left(\frac{(\omega-1)n(1+\beta)}{2\beta(1+n)} + e_{ss}\right) && (62)
\end{aligned}
$$

$$
\begin{aligned}
e_{ss} &= \frac{4\omega}{27}\frac{\gamma\beta^2\theta^3}{(1+n)^2(1+\beta)^2}\frac{x}{1-x}\left(\frac{(\omega-1)n(1+\beta)}{2\beta(1+n)} + e_{ss}\right) && (63)\\
e_{ss}\left(1-\frac{4\omega}{27}\frac{\gamma\beta^2\theta^3}{(1+n)^2(1+\beta)^2}\frac{x}{1-x}\right) &= \frac{4\omega}{27}\frac{\gamma\beta^2\theta^3}{(1+n)^3(1+\beta)^2}\frac{x}{1-x}\left(\frac{(\omega-1)n(1+\beta)}{2\beta(1+n)} + e_{ss}\right) && (64)\\
e_{ss}\left(\frac{27(1+n)(1+\beta)^2(1-x) - 4\omega\gamma\beta^2\theta^3 x}{27(1+n)^2(1+\beta)^2(1-x)}\right) &= \frac{4(\omega-1)\gamma\beta\theta^3 xn}{54(1+n)^3(1+\beta)(1-x)} && (65)\\
e_{ss} &= \frac{2(\omega-1)\gamma\beta\theta^3 xn(1+\beta)}{(1+n)[27(1+n)^2(1+\beta)^2(1-x) - 4\omega\gamma\beta^2\theta^3 x]} && (66)
\end{aligned}
$$

Retrieve i_{ss} in order to get k_{ss}:

$$
\begin{aligned}
i_{ss} &= \frac{x}{1-x}\left(\frac{(\omega-1)n(1+\beta)}{2\beta(1+n)} + \frac{2(\omega-1)\gamma\beta\theta^3 xn(1+\beta)}{(1+n)[27(1+n)^2(1+\beta)^2(1-x) - 4\omega\gamma\beta^2\theta^3 x]}\right) && (67)\\
&= \frac{x}{(1-x)(1+n)}\frac{(\omega-1)n(1+\beta)[27(1+n)^2(1+\beta)^2(1-x) - 4\omega\gamma\beta^2\theta^3 x] + 4(\omega-1)\beta^2\gamma\theta^3 xn(1+\beta)}{2\beta[27(1+n)^2(1+\beta)^2(1-x) - 4\omega\gamma\beta^2\theta^3 x]} && (68)\\
&= x\frac{27(\omega-1)n(1+n)^2(1+\beta)^3(1-x) + 4(\omega-1)\beta^2\gamma\theta^3 xn(1+\beta)(1-\omega)}{2\beta(1-x)(1+n)[27(1+n)^2(1+\beta)^2(1-x) - 4\omega\gamma\beta^2\theta^3 x]} && (69)\\
&= \frac{(\omega-1)xn(1+\beta)[27(1+n)^2(1+\beta)^2(1-x) - 4(\omega-1)\beta^2\gamma\theta^3 x]}{2\beta(1-x)(1+n)[27(1+n)^2(1+\beta)^2(1-x) - 4\omega\gamma\beta^2\theta^3 x]} && (70)\\
&= \frac{(\omega-1)xn(1+\beta)}{2\beta(1-x)(1+n)} + \frac{4(\omega-1)nx^2\beta^2\gamma\theta^3(1+\beta)}{2\beta(1-x)(1+n)[27(1+n)^2(1+\beta)^2(1-x) - 4\omega\gamma\beta^2\theta^3 x]} && (71)\\
&= \frac{(\omega-1)xn(1+\beta)}{(1-x)(1+n)}\left(\frac{1}{2\beta} + \frac{2x\beta\gamma\theta^3}{27(1+n)^2(1+\beta)^2(1-x) - 4\omega\gamma\beta^2\theta^3 x}\right) && (72)\\
& && (73)
\end{aligned}
$$

Therefore:

$$
\begin{aligned}
k_{ss} &= \left[\frac{2\beta\theta}{3n(1+\beta)}\frac{(\omega-1)xn(1+\beta)}{(1-x)(1+n)}\left(\frac{1}{2\beta} + \frac{2x\beta\gamma\theta^3}{27(1+n)^2(1+\beta)^2(1-x) - 4\omega\gamma\beta^2\theta^3 x}\right)\right]^{\frac{3}{2}} && (74)\\
&= \left[\frac{2x\beta\theta(\omega-1)}{3(1-x)(1+n)}\left(\frac{1}{2\beta} + \frac{2x\beta\gamma\theta^3}{27(1+n)^2(1+\beta)^2(1-x) - 4\omega\gamma\beta^2\theta^3 x}\right)\right]^{\frac{3}{2}} && (75)
\end{aligned}
$$

Note that in the steady state, the condition not to observe a zero-capital equilibrium is:

$$(1+n)\frac{b_{ss}}{b_{ss}} < \frac{\alpha n(1+\beta)}{\beta(1-\alpha)i_{ss}} + 1 + \varepsilon \tag{76}$$

$$n < \frac{n(1+\beta)}{2\beta}\frac{(1-x)(1+n)}{(\omega-1)xn(1+\beta)}\frac{2\beta[27(1+n)^2(1+\beta)^2(1-x)-4\omega\gamma\beta^2\theta^3x]}{[27(1+n)^2(1+\beta)^2(1-x)-4(\omega-1)\gamma\beta^2\theta^3x]} + \varepsilon \tag{77}$$

$$\frac{(1-x)(1+n)[27(1+n)^2(1+\beta)^2(1-x)-4\omega\gamma\beta^2\theta^3x]}{(\omega-1)xn[27(1+n)^2(1+\beta)^2(1-x)-4(\omega-1)\gamma\beta^2\theta^3x]} > 1 - \frac{\varepsilon}{n} \tag{78}$$

$$\frac{(1-x)(1+n)[27(1+n)^2(1+\beta)^2(1-x)-4\omega\gamma\beta^2\theta^3x]}{(\omega-1)xn[27(1+n)^2(1+\beta)^2(1-x)-4(\omega-1)\gamma\beta^2\theta^3x]} > 1 - \frac{\varepsilon}{n} \tag{79}$$

Steady State with $b_{ss} = 0$

Now, look at the steady-state equilibrium in the case in which $b_{ss} = 0$. This occurs when the asset loses all of its value and/or in the limit for $t \to \infty$ when B is constant and population growth is positive.

$$b_{ss} = 0 \tag{80}$$

Rewrite i_{ss} and e_{ss} as follows and solve the system.

$$i_{ss} = x(e_{ss} + a_{ss}) = \frac{xe_{ss}}{1-x} \tag{81}$$

$$e_{ss} = \frac{\gamma a_t^2 r_{ss} i_{ss}}{1+n} \tag{82}$$

$$e_{ss} = \frac{\gamma}{1+n}\left(\frac{\beta(1-\alpha)\theta i_{ss}^{\alpha}}{(1+\beta)n^{\alpha}}\right)^{\frac{2}{1-\alpha}}\frac{\alpha n(1+\beta)}{\beta(1-\alpha)i_{ss}}i_{ss} \tag{83}$$

$$= \frac{\gamma\alpha\beta^{\frac{2-1+\alpha}{1-\alpha}}(1-\alpha)^{\frac{2-1+\alpha}{1-\alpha}}\theta^{\frac{2}{1-\alpha}}i_{ss}^{\frac{2\alpha}{1-\alpha}}}{(1+n)(1+\beta)^{\frac{2-1+\alpha}{1-\alpha}}n^{\frac{2\alpha-1+\alpha}{1-\alpha}}} \tag{84}$$

$$= \frac{\gamma\alpha\beta^{\frac{1+\alpha}{1-\alpha}}(1-\alpha)^{\frac{1+\alpha}{1-\alpha}}\theta^{\frac{2}{1-\alpha}}x^{\frac{2\alpha}{1-\alpha}}}{(1+n)(1+\beta)^{\frac{1+\alpha}{1-\alpha}}n^{\frac{3\alpha-1}{1-\alpha}}(1-x)^{\frac{2\alpha}{1-\alpha}}}e^{\frac{2\alpha}{1-\alpha}} \tag{85}$$

$$e_{ss}^{1-\alpha-2\alpha} = \left(\frac{\gamma\alpha}{1+n}\right)^{1-\alpha} n^{1-3\alpha}\left(\frac{\beta(1-\alpha)}{1+\beta}\right)^{1+\alpha}\left(\frac{\theta x^{\alpha}}{(1-x)^{\alpha}}\right)^2 \tag{86}$$

$$e_{ss} = n\left[\left(\frac{\gamma\alpha}{1+n}\right)^{1-\alpha}\left(\frac{\beta(1-\alpha)}{1+\beta}\right)^{1+\alpha}\left(\frac{\theta x^{\alpha}}{(1-x)^{\alpha}}\right)^2\right]^{\frac{1}{1-3\alpha}} \tag{87}$$

$$k_{ss} = \left[\frac{\beta(1-\alpha)xn}{n(1+\beta)(1-x)}\left(\frac{\gamma\alpha}{1+n}\right)^{\frac{1-\alpha}{1-3\alpha}}\left(\frac{\beta(1-\alpha)}{1+\beta}\right)^{\frac{1+\alpha}{1-3\alpha}}\left(\frac{\theta x^{\alpha}}{(1-x)^{\alpha}}\right)^{\frac{2}{1-3\alpha}}\right]^{\frac{1}{1-\alpha}} \tag{88}$$

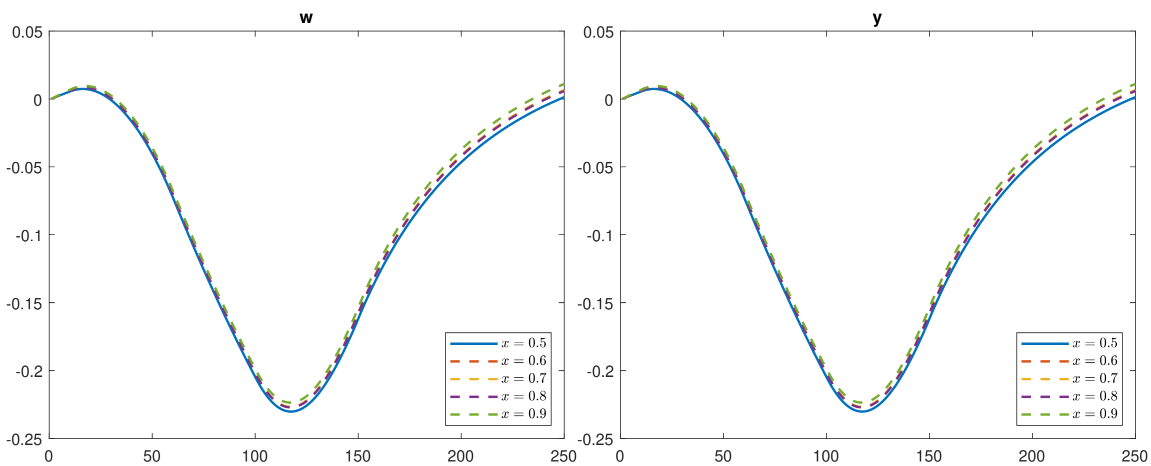
$$= \left[\left(\frac{\gamma\alpha}{1+n}\right)^{\frac{1-\alpha}{1-3\alpha}}\left(\frac{\beta(1-\alpha)}{1+\beta}\right)^{\frac{2(1-\alpha)}{1-3\alpha}}\theta^{\frac{3(1-\alpha)}{1-3\alpha}}\left(\frac{x}{1-x}\right)^{\frac{1-\alpha}{1-3\alpha}}\right]^{\frac{1}{1-\alpha}} \tag{89}$$

$$= \left[\frac{\gamma\alpha x\theta^3}{(1+n)(1-x)}\left(\frac{\beta(1-\alpha)}{1+\beta}\right)^2\right]^{\frac{1}{1-3\alpha}} \tag{90}$$

5.2 Figures and Tables

Parameter	Value
δ (capital depreciation)	0
n (population growth)	0.02
β (discount rate)	0.9
α (capital share of output)	1/3

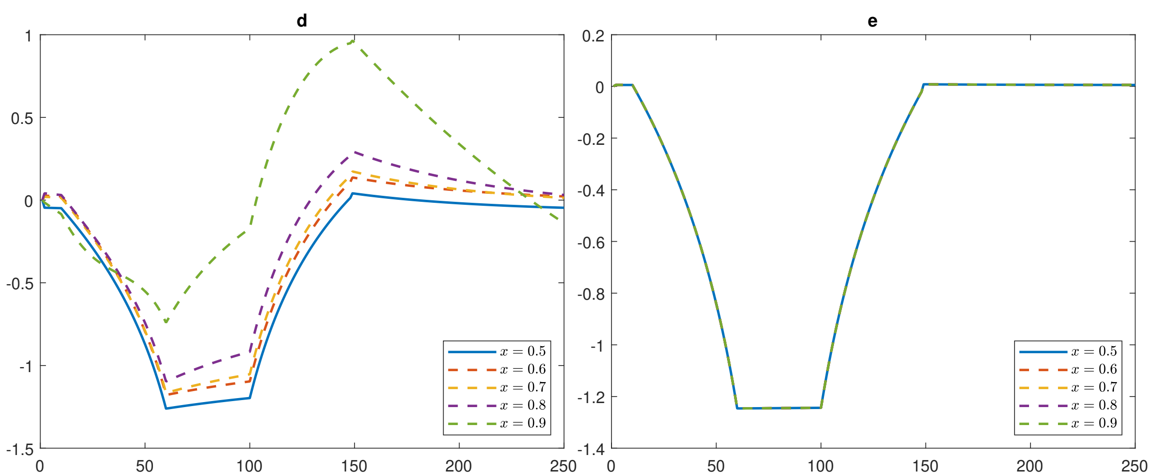
Table 2: Parameter values



(a) Wages

(b) Output

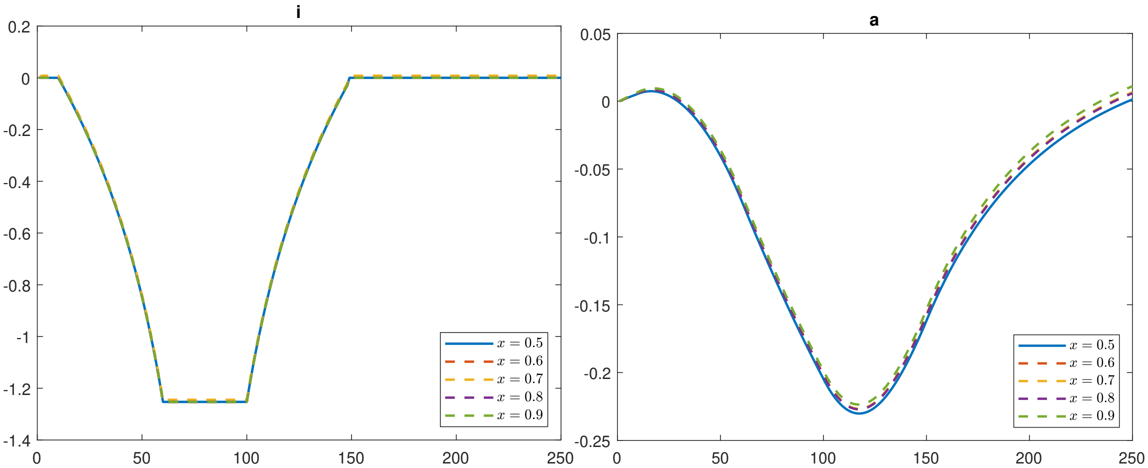
Figure 8: Wages and output (log-deviations from SS) for different levels of x.



(a) Dividends

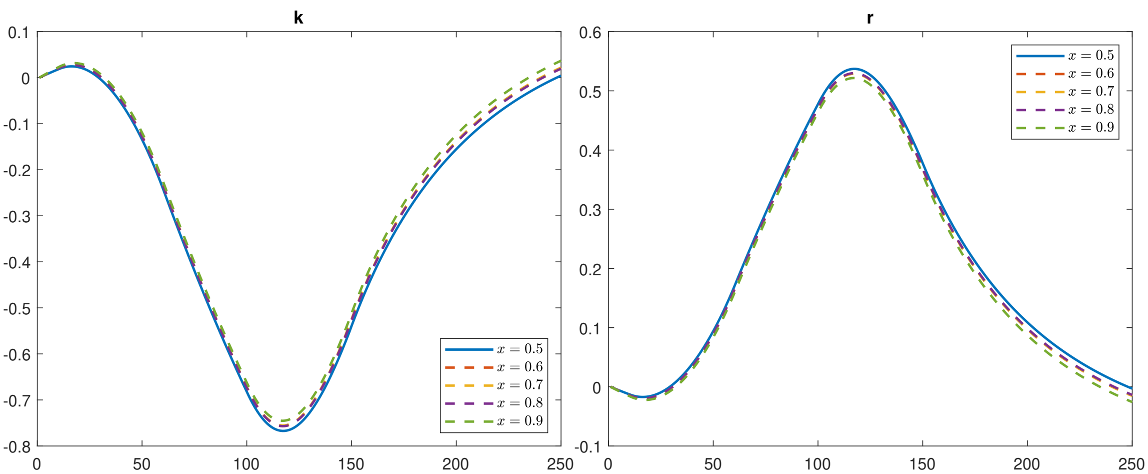
(b) Bank's equity

Figure 9: Consumption for Young and Old Generations for Different Levels of x.



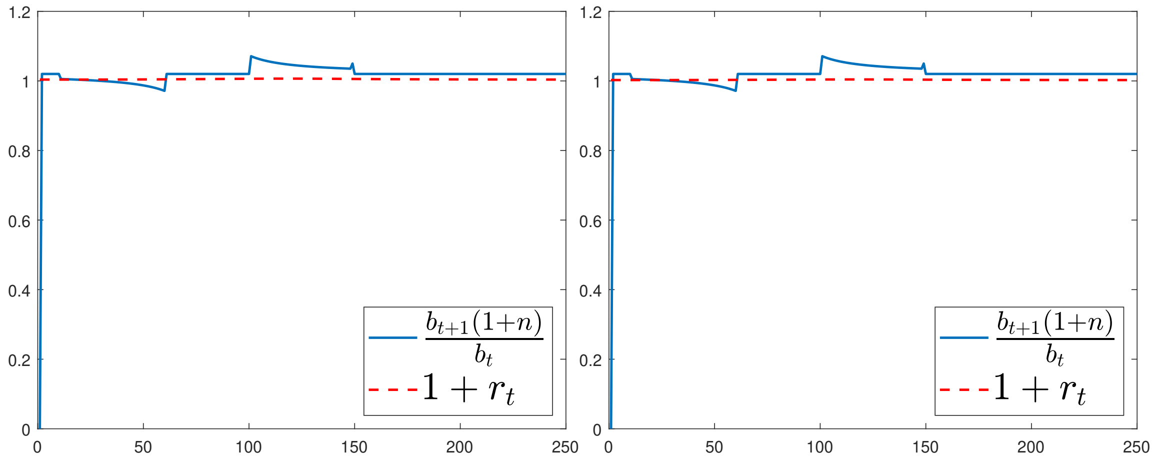
(a) Investment in capital goods (b) Assets

Figure 10: Investment in capital goods and assets (log-deviations from SS) for different levels of x.



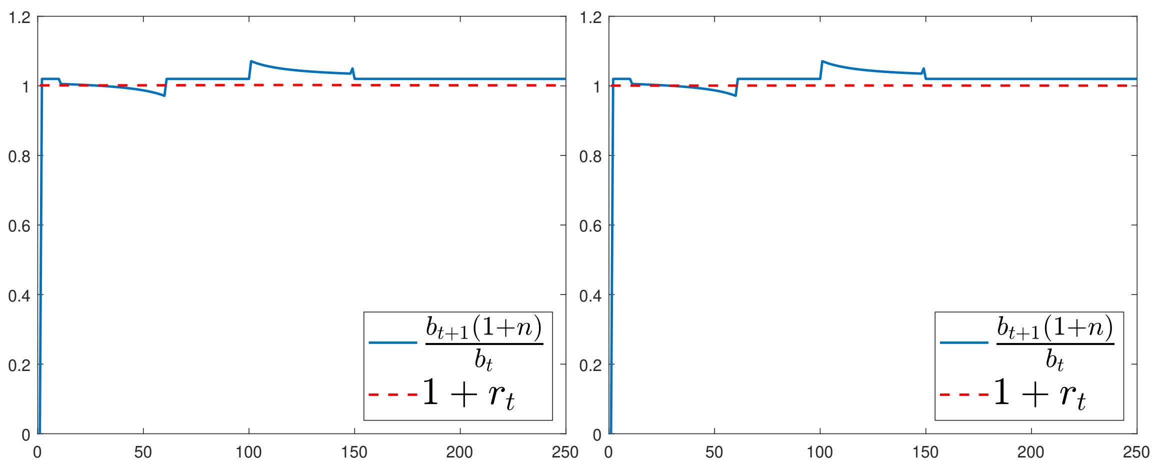
(a) Capital (b) Capital rate of return

Figure 11: Capital and capital return (log-deviations from SS) for different levels of x.



(a) $x = 0.6$ (b) $x = 0.7$

Figure 12: Local Indifference Condition for $x = 0.6$ and $x = 0.7$.



(a) $x = 0.8$ (b) $x = 0.9$

Figure 13: Local Indifference Condition for $x = 0.8$ and $x = 0.9$.