



**Master's Degree in Economics and Finance
Finance Program**

**Does Macro-Financial Information
matter for Growth at Risk forecasting?**

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Abstract

In order to analyse whether financial conditions are relevant downside risk predictors for the 5% Growth at Risk conditional quantile, we propose a Dynamic Factor-GARCH Model, comparing it to the two most relevant approaches in the literature. We conduct an in-sample backtesting exercise and an out-of sample forecasting analysis, including the national financial conditions index, term structure and housing prices for 17 European countries and the United states, as down side risk predictors. If anything, we find evidence of significant predicting power of financial conditions, which, if exploited correctly, increases in relevance in times of extraordinary financial distress.

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1 Introduction

In their relatively recent paper Adrian, Boyarchenko, and Giannone (2016) developed the concept of Growth-at-Risk (GaR), a measure of downside risk that quantifies the worst case scenario for the GDP growth rate given a specific confidence level; analogous to the Value-at-Risk used in Risk Management, both methodologies share the statistical framework for in-sample backtesting and out-of-sample forecasting analysis. Eventually, the International Monetary Fund (IMF) popularized this measure of macroeconomic risk (Adrian, Grinberg, et al., 2018, Prasad et al., 2019, and Adams et al., 2021) by linking national financial conditions to the distribution of GDP growth through the use of quantile regressions, arousing the interest of public institutions, Central Banks and practitioners (Plagborg-Møller et al., 2020, Brownlees and Souza, 2021, and Delle Monache, De Polis, and Petrella, 2021 among others).

More generally, the Growth-at-Risk can be placed in the strand of literature studying the relationship between finance and macroeconomics. Starting from DSGE models with financial accelerators á la Bernanke, Gertler, and Gilchrist (1999), as the ones proposed by Christiano, Motto, and Rostagno (2014) and Del Negro, Giannoni, and Schorfheide (2015), to SVAR empirical analyses studying the role of financial frictions on the real economy (Aysun, Brady, and Honig, 2013, and Gertler and Karadi, 2015 for instance). During recent years, the idea that financial variables might be one of the main determinants of business cycle fluctuations has been prevailing more and more. This exact rationale underpinned the work of Adrian, Boyarchenko, and Giannone (2016), in which the authors modelled the five percent distribution region of GDP growth rates by using a set of quarterly financial variables produced by the IMF. However, Brownlees and Souza (2021) showed that simply fitting a AR(1)-GARCH(1,1) model¹ within a framework of 24 OECD countries produces better in-sample and out-of-sample results when compared to the quantile regression approach. Should we therefore assume that financial conditions do not actually have any predicting power regarding the density forecast for GDP growth rates?

Our work aims to delve deeper into this issue by investigating whether financial conditions are good predictors for the left-tail distribution regions of GDP growth rates, i.e. for the 5% GaR conditional quantile, or not. While the shortcomings of Adrian, Boyarchenko, and Giannone (2016)'s quantile regression were pointed out, it is important to distinguish whether those issues are to be blamed on the model or on the lack of significance of the financial variables in terms of GDP growth rate predictability. Therefore, we are

¹Bollerslev, Engle, and Nelson (1994), Bauwens and Laurent (2005), and Silvennoinen and Teräsvirta (2009) for a general review regarding this class of models.

going to model the finance-macro relationship otherwise, by proposing and implementing a Dynamic Factor-GARCH Model, combining the strengths of macro-econometrics and financial time series workhorse models. Our idea is to map financial conditions into GDP growth through the use of a different estimation procedure based on Principal Component Analysis; consequently, we will work only with European countries in order to retrieve and interpret the latent common factors as measures of systemic risk and aggregate financial distress in Europe.

This paper consists of two separate analyses in which we compare the performance of the three aforementioned models: the quantile regression of Adrian, Boyarchenko, and Giannone (2016), the model of Brownlees and Souza (2021), and ours. First, we will conduct an in-sample Growth-at-Risk backtest based on several hypothesis tests and other econometric measures. Subsequently, we will design an out-of-sample forecasting exercise with the relative density forecast evaluation.

The rest of the paper is organized as follows: in section 2 we will provide evidence about common trends in the dynamics of national financial conditions for the European countries, justifying the implementation of a Dynamic Factor Model. In section 3 we thoroughly explore the econometric frameworks of the three estimation procedures and will present the hypothesis tests for the two empirical analyses, the results of which will be presented in section 4. Lastly, a final remark will close our work in the fifth section. For the sake of clarity, more extensive tables and figures are presented in two separate sections at the end of the text, before the appendix.

2 What is the Data Telling Us?

Our work is inspired by the rich environment of empirical findings and estimation procedures around Growth-at-Risk coming mainly from the works of Adrian, Boyarchenko, and Giannone (2016), Plagborg-Møller et al. (2020) as well as Brownlees and Souza (2021). Instead of imposing a model a priori we will follow a data driven approach by first studying the dynamics of our financial variables. In the following we are going to introduce the dataset we used and justify the selection of our sample. Are there common trends? Is there evidence of a European convergence among financial conditions? These are the two main questions that we are going to answer in this second section, in order to properly map the financial variables into the GDP growth rates.

2.1 Data Exploration

We conduct our analysis based on the data set explored by Brownlees and Souza (2021), focusing on a total of 18², out of the 24, OECD countries: 17, of which are located in the geographical Europe area plus the United States as a benchmark and proxy for global financial conditions. As previously mentioned, focusing only on European countries will help us catch and exploit the possible presence of spill-over effects, common trends and common co-movements with respect to business cycle fluctuations.

Since the Quantile Regression of Adrian, Boyarchenko, and Giannone (2016) relies on a set of financial conditions as down side risk predictors, we choose to base our data selection on the results of Brownlees and Souza (2021), who have implemented a full-fledged statistical analysis to assess the significance of several financial variables and other time series on the GDP growth rates. Contrasting to Brownlees and Souza (2021) we focus on the country-specific, rather than the global predictors (e.g. the Geopolitical Risk Index of Caldara and Iacoviello (2018)), to be able to capture European-specific effects of financial variables. This - also in conformity with the IMF method - leaves us with the national financial conditions index (NFCI), term spread (TS), housing prices (HP) and the available GDP growth rates as our set of financial down side risk predictors.

It is important to mention that Brownlees and Souza (2021) used the NFCI index made available by the IMF to construct a *"measure of Global NFCI as the average of the country-specific available observations at each point in time"*³ for imputation purposes: the authors

²Austria (AUT), Belgium (BEL), Denmark (DNK), Finland (FIN), France (FRA), Germany (DEU), Greece (GRC), Iceland (ISL), Ireland (IRL), Italy (ITA), Luxembourg (LUX), Norway (NOR), Portugal (PRT), Spain (ESP), Sweden (SWE), Switzerland (CHE), the U.K. (GBR) and the U.S.A. (USA).

³Brownlees and Souza (2021), Online appendix, page 8 - <https://ars.els-cdn.com/content/image/1-s2.0-S0304393220301288-mm1.pdf>

assign their Global NFCI to all countries of their sample that do not have the NFCI published⁴. In our specific selection this lack of data concerns Austria, Belgium, Denmark, Finland, Greece, Iceland, Ireland, Luxembourg, Norway and Portugal. Partially missing data in our sample has been imputed for Italy, Spain, Sweden and Switzerland. The complete IMF series was used for France, Germany, the United Kingdom and the United States.

Regarding the second financial variable, Brownlees and Souza (2021) construct the term spread by subtracting short-term interest rates from long-term interest rates obtained from the OECD website. Similar to the NFCI, the authors create a global measure to impute missing data.

Lastly, for the housing prices, the authors use BIS data on property price statistics or data from each country's statistical agency whenever possible. Again, a measure of global property prices is constructed and used for imputation. Given that there was no data available for Iceland, the global measure was used instead.

The next step in our analysis consists of performing a Principal Component Analysis (PCA) of these financial variables to understand whether common dynamics and latent factors are present and, if they are, how much of the volatility of our series they are able to explain. Before presenting our results, we will discuss the statistical details of the PCA estimation procedure we implemented in the next paragraph.

2.2 Principal Component Analysis

Our starting point is the specification of a vector of normalized endogenous variables X_t , which are GDP growth rates, NFCI, TS and HP in our case. This vector could be decomposed as follows:

$$X_t = \Lambda F_t + \varepsilon_t \quad (1)$$

in matrix notation:

$$X_t = \begin{bmatrix} X_{1t} \\ X_{2t} \\ \vdots \\ X_{Nt} \end{bmatrix} \Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \dots & \Lambda_{1r} \\ \Lambda_{21} & \Lambda_{22} & \dots & \Lambda_{2r} \\ \vdots & \vdots & \vdots & \vdots \\ \Lambda_{N1} & \Lambda_{N2} & \dots & \Lambda_{Nr} \end{bmatrix} F_t = \begin{bmatrix} F_{1t} \\ F_{2t} \\ \vdots \\ F_{rt} \end{bmatrix} \varepsilon_t = \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \vdots \\ \epsilon_{Nt} \end{bmatrix} \quad (2)$$

where Λ is the $(N \times r)$ matrix of factor loadings which maps the r -dimensional column vector of common factors to the $(N \times 1)$ vector of endogenous variables, while ε is the N -dimensional column vector of idiosyncratic disturbances. However, only the vector X_t

⁴This procedure maintains the original series scale properties

is observable - this is the reason why we are going to explain the estimation procedure that we implement for retrieving the time series of latent factors, also called principal components. It is important to mention that ΛF_t could be estimated jointly only up to the choice of a suitable rotation matrix.

Firstly, we assume the factors F_t to be known in order to construct the following minimization problem:

$$\begin{cases} \min_{\{F_1, \dots, F_T; \Lambda\}} \sum_{t=1}^T (X_t - \Lambda F_t)'(X_t - \Lambda F_t) \\ s.t. \quad N^{-1} \Lambda' \Lambda = I_r \end{cases} \quad (3)$$

which solution, after taking the first derivative with respect to the vector of the factor, is easily computed by OLS:

$$F = (\Lambda' \Lambda)^{-1} (\Lambda' X) \quad . \quad (4)$$

Since the matrix of factor loadings Λ is not observed, we plug this result back into the objective function:

$$\min_{\{\Lambda\}} -NT^{-1} \sum_{t=1}^T (X_t' - \Lambda(\Lambda' \Lambda)^{-1} \Lambda' X_t)' (X_t' - \Lambda(\Lambda' \Lambda)^{-1} \Lambda' X_t) \quad (5)$$

Clearly, $\Lambda(\Lambda' \Lambda)^{-1} \Lambda'$ is the projection matrix P , properties of which are:

$$P^2 = P \quad \text{and} \quad P' = P \quad . \quad (6)$$

After some algebra manipulations which involve these two properties, the constrained optimization problem becomes:

$$\begin{cases} \max_{\{\Lambda\}} NT^{-1} \sum_{t=1}^T X_t' (\Lambda(\Lambda' \Lambda)^{-1} \Lambda') X_t \\ s.t. \quad N^{-1} \Lambda' \Lambda = I_r \end{cases} \quad (7)$$

which is equivalent to:

$$\begin{cases} \max_{\{\Lambda\}} \sum_{t=1}^T \text{trace}\{\Lambda' \Sigma_{XX} \Lambda\} \\ s.t. \quad N^{-1} \Lambda' \Lambda = I_r \end{cases} \quad (8)$$

Just for the sake of clarity, $\Lambda = |\Lambda_1 \dots \Lambda_r|$ is the vector of columns of the factor loading matrix and Σ_{XX} is the variance/covariance matrix of the endogenous variables. The strategy that we implement to solve this system is, first, to rewrite the system in the

following form:

$$\begin{cases} \max_{\{\Lambda_1, \dots, \Lambda_r\}} & \sum_{i=1}^r \Lambda_i \Sigma_{XX} \Lambda_i' \\ \text{s.t.} & N^{-1} \Lambda' \Lambda = I_r \end{cases} \quad (9)$$

and second, to differentiate the lagrangian function with respect to each column vector Λ_i and to put it equal to zero, which leads to:

$$\frac{\partial(\Lambda_i' \Sigma_{XX} \Lambda_i - \lambda(\Lambda_i' \Lambda_i - 1))}{\partial \Lambda_i} = 0 \quad \text{that is:} \quad \Sigma_{XX} \Lambda_i - \lambda \Lambda_i = 0 \quad (10)$$

The solution is straightforward, with Λ_i that is the eigenvector associated with the i^{th} -eigenvalue λ of the variance covariance matrix of the endogenous variables:

$$\Sigma_{XX} = \Lambda \Sigma_{FF} \Lambda' + \Sigma_{\varepsilon\varepsilon} \quad (11)$$

By substituting $\hat{\Lambda} = [\hat{\Lambda}_1 \dots \hat{\Lambda}_r]$ back into equation (4), we are able to compute the latent factors:

$$\hat{F}_t = (\hat{\Lambda}' \hat{\Lambda})^{-1} (\hat{\Lambda}' X_t) = \frac{\hat{\Lambda}' X_t}{N} \quad (12)$$

The choice regarding the number of principal components to estimate using the PCA is arbitrary⁵. In our case we will base this decision on the percentage of variance explained criterion. In the next paragraph, we will discuss the empirical evidence coming from the Principal Component Analysis, which will be the ratio underpinning the implementation of our Dynamic Factor-GARCH Model.

2.3 Convergence of Financial Time Series Processes

After obtaining our data set and having explained the statistical framework of the Principal Component Analysis, we perform a visual analysis of the financial variable time series as a first step, to "eyeball" the possible presence of convergence effects in terms of national financial conditions and macroeconomic outcomes at European level. In figure 1 we observe that almost all of our time series exhibit some degree of convergence especially after the 1990's. This might be due to political and financial collusion in the Euro area during that period - however, we have to keep the imputations of missing data performed by Brownlees and Souza (2021) in mind. As far as it concerns the NFCI, figure 1a, even if 10 out of 18 of our countries' time series are entirely based on the measure of global NFCI

⁵During the last years the use of the Mean Squared Reconstruction Error computed from mapping the latent factors into the original dataset has become increasingly popular, especially with the implementation of deep learning dimensionality reduction techniques such as Auto-Encoder.

constructed by the authors ⁶, we actually observe that the series of the remaining countries also exhibit strong convergence after the virtual formation of the Euro in 1999. Regarding the term structure, figure 1b, the co-integration among different European states is even stronger than the NFCI's; in fact, the term structures did not only follow the same fluctuations but rather also had approximately the same absolute values in movements over the entire time period considered. The third variable considered is housing prices as seen in figure 1c: in that case, even if we observe a higher degree of heterogeneity among European countries, all series show the same peaks and troughs. Lastly, in figure 1d we plot the dynamics of the GDP growth rates, which exhibit a stronger convergence in terms of business cycle fluctuations especially after the launch of the Euro.

Moreover, we also focus our attention on the correlation among the predictors, NFCI, TS and HP and the past realization of GDP growth rates, which we selected based on the analysis of Brownlees and Souza (2021), proving their strong significance for predicting national GDP growth rates. Plotting the correlation matrix (figure 2), we observe a very strong correlation among the variables of our data set. In particular, the GDP growth rates turn out to be negatively correlated with the NFCI and positively correlated with term structure, housing price and with the economic outcomes of the other European countries. The macroeconomic outcomes also display some degrees of correlation with the financial conditions of the other countries.

After this initial examination, we construct our own Principal Component Analysis in order to assess the predicting power of the latent factors in terms of percentage of variance explained, performing both, a static and a rolling window PCA. In doing so, we observe that the first five principal components are able to explain a good 75% of the volatility of all the series through the entire time period (figure 3a). Therefore, we will precisely rely on those five latent factors for our Dynamic Factor-GARCH Model to break down the so called curse of dimensionality, in order to extrapolate all the relevant information from the cross-sectional dimensions of our panel of time series data. In figure 3b, we perform a dynamic PCA for those five latent factors by estimating the matrix of factor loadings Λ relying on a rolling window approach of 50 quarters, to show the dynamics of the variability explained of said latent factors over time. The first and second factors display countercyclical behaviour, especially noticeable during the 2008 financial crisis with the first factor increasing the variance explained and the second one decreasing. It seems that the first factor could be understood as some kind of indicator for financial stress while the second one might exhibit financial stability conditions (also noticeable during the European co-integration period during the 1990s and during the introduction

⁶This fact should be considered also during the second step of this data analysis, regarding the percentage of variance explained by the factors.

of the Euro). Factors three to five exhibit far smoother behaviour and they only add a marginal, but relevant, amount of variance explained.

To conclude, the results arising from the Principal Component Analysis underpin our choice of using five common factors to model the distribution of the GDP growth rates. In the next section, after introducing the GaR models of Adrian, Boyarchenko, and Giannone (2016) and Brownlees and Souza (2021), we will build our model explaining the econometric procedure to exploit these common components.

3 Growth-at-Risk: Models and Empirical Evaluation

In the last years economists and important institutions have been focusing their attention on modelling and forecasting the marginal and the joint distribution of GDP growth rates in order to measure the downside risk associated with the extreme events of the 5% conditional quantile, the so called Growth-at-Risk. Given the GDP growth rate Y_{it} of the i^{th} country, the h -step ahead Growth-at-Risk is defined as the maximum loss that could be observed at α confidence level:

$$\mathbb{P}(Y_{it+h} \leq GaR_{it+h|t}^\alpha) = \alpha \quad \text{with: } \alpha = 0.05 \quad (13)$$

id est the $GaR_{it+h|t}^\alpha$ should contain the real GDP growth rates α of the time. In other words, the Growth-at-Risk is the α -quantile of the conditional distribution of the gross domestic product:

$$GaR_{it+h|t}^\alpha = F_{Y_{it+h}}^{-1}(\alpha) \quad (14)$$

Consequently, one of the most important questions arising from this branch of literature concerns the understanding of what variables could model these distribution regions and what model could exploit their predictive power. Starting with the paper of Adrian, Boyarchenko, and Giannone (2016), financial variables were appointed as the main predictor for the downside risk of the conditional distribution, while Brownlees and Souza (2021) proved that by simply modelling the variance of an autoregressive process, without including any other variables, they could obtain better results both in sample and out of sample. Should we definitely discard the use of financial conditions? Is their role negligible in predicting the lowest quantiles of the distributions of the GDP growth rates? In our work we will answer these questions which are part of an important ongoing debate regarding the connection between Finance and Macroeconomics.

In this section we are going to introduce three different models embedding distinct assumptions on the data generating process and contrasting views on the main determinants of GDP conditional distributions; after, we will describe the two empirical analyses conducted, the in-sample backtesting and the out-of-sample forecasting, in order to find an answer to the seeming disconnect between finance and macro. The first model is the one proposed by Adrian, Boyarchenko, and Giannone (2016) and popularized by the IMF, the second one comes from the important paper of Brownlees and Souza (2021) and the last one is the model that we developed, combining a Dynamic Factor Model with a GARCH class of model for the conditional volatility. Our DF-GARCH Model represents an unicum in the econometric literature: for the best of our knowledge, in the past only Alessi, Barigozzi, and Capasso (2007) combined these two estimation procedures but just imposing a conditional heteroskedastic process for the dynamic common factors.

3.1 Downside-Risk Through Quantile Regression

The first model we are analyzing for Growth-at-Risk estimation was proposed by Adrian, Boyarchenko, and Giannone (2016), and thoroughly analyzed in Adrian, Grinberg, et al. (2018) and Prasad et al. (2019). The authors argued that deteriorating financial conditions generate the observed growth vulnerability dynamics. Moreover, they provided evidence about how the fifth percent quantile (the $GaR_{it+h|t}^\alpha$) of the distribution of future GDP growth rates exhibits strong variation as a function of current financial conditions. Adrian, Boyarchenko, and Giannone (2016) employed a quantile regression approach (Koenker and Bassett Jr, 1978) for the estimation of the Growth-at-Risk. Starting from the minimization of the quantile weighted absolute value of errors under the Linlin loss function (Christoffersen and Diebold, 1997):

$$\min_{\beta_\alpha \in \mathbb{R}^k} \sum_{t=1}^{T-h} \alpha |Y_{it+h} - X_{it}\beta^\alpha| \mathbb{1}_{Y_{it+h} \geq X_{it}\beta_\alpha} + (1 - \alpha) |Y_{it+h} - X_{it}\beta^\alpha| \mathbb{1}_{Y_{it+h} \leq X_{it}\beta_\alpha} \quad (15)$$

where X_t is the vector of financial predictors at time t , the Growth-at-Risk is obtained as the fitted value from that regression:

$$GaR_{it+h|t}^\alpha = \mathbb{Q}_{Y_{it+h}|X_t}(\alpha|X_{it}) = X_{it}\hat{\beta}^\alpha \quad (16)$$

Given the information set \mathcal{F}_t , producing the h -step ahead forecast is straightforward since it only concerns the estimation of β^α given a different time length. As far as it concerns the matrix of predictors, X_{it} , we employ NFCI, Housing Price, Term Structure and GDP growth rates available at time t .

This estimation procedure is based on directly modelling the desired distribution region relying on the idea that financial variables are the main determinants of the GDP downside shortfalls. Two questions arise at this point, concerning whether (1) this is the right approach to exploit the predictive power of financial conditions and (2) whether these variables are good predictors for GDP distributions or not.

3.2 Brownlees and Souza (2021) Approach

In their paper Brownlees and Souza (2021) estimated the Growth-at-Risk using a non-parametric approach. Given the stochastic process $\{T_t^i\}_{t \in \mathbb{Y}^+}$ defined as:

$$Y_{it+1} = \mu_{it+1|t} + \sqrt{\sigma_{it+1|t}^2} Z_{it+1} \quad \text{with: } Z_{it+1} \sim iid \mathcal{D}_{Z_i}(\mu, \sigma) \quad (17)$$

the authors modelled two processes for the conditional mean and the conditional variance of the GDP growth rates and proceeded to estimate the distribution $\mathcal{D}_{Z_i}(\mu, \sigma)$

non-parametrically, using the standardized residuals $\{\widehat{Z}_{it-H}, \dots, \widehat{Z}_{i1}\}_{i=1}^N$. Lastly, inference about the α -quantile of the GDP growth rates was carried out by inverting the distribution $Y_{it+1}|\mathcal{F}_t \sim \mathbb{F}_{Y_{it+1}}(\mu_{it+1|t}, \sigma_{it+1|t}^2)$:

$$\text{GaR}_{it+h|t}^\alpha = \mathbb{F}_{Y_{it+h}}^{-1}(\alpha) = \mu_{it+h|t} + \sqrt{\sigma_{it+h|t}^2} \mathcal{D}_{Z_i}^{-1}(\alpha) . \quad (18)$$

For the conditional mean $\mu_{it+h|t}$ they fitted an autoregressive process of order one:

$$Y_{it} = \phi_{i0} + \phi_{i1}Y_{it-1} + \varepsilon_{it} \quad \text{with: } \varepsilon \sim N(0, 1) \quad (19)$$

while for the conditional variance $\sigma_{it+1|t}^2$, they modelled different GARCH-type specifications:

- GARCH(1,1): $\sigma_{it+1|t}^2 = \omega_i + \alpha_i \varepsilon_{it}^2 + \beta_i \sigma_{it}^2$;
- Panel-GARCH(1,1): $\sigma_{it+1|t}^2 = \omega_i + \alpha \varepsilon_{it}^2 + \beta \sigma_{it}^2$;
- TAR(1,1): $\sigma_{it+1|t}^2 = \omega_i + \alpha_i \varepsilon_{it}^2 + \gamma_i \varepsilon_{it}^2 \mathbf{1}_{\{\varepsilon_{it} < 0\}} + \beta_i \sigma_{it}^2$.⁷

In their paper Brownlees and Souza (2021) showed that better results could be obtained with respect to the quantile regression approach of Adrian, Boyarchenko, and Giannone (2016) both in-sample and out-of-sample, by simply modelling the conditional variance of the GDP growth rates. Should we conclude that national financial conditions are not good predictors for the distribution of GDP growth rates? Surely the IMF estimation procedure faces some shortcomings, but this does not directly imply that financial variables do not play any role in predicting the downside risk associated to GDP growth rates. Hence, we propose a model which takes the set of financial variables described in the previous section into account and models the conditional variance of the macroeconomic time series at the same time. Our aim is not only trying to develop a more accurate model for the Growth-at-Risk, but also researching the relationship between finance and macroeconomics, studied by academics and practitioners for decades.

3.3 Our Approach: Dynamic Factor-GARCH Model

One of the most important empirical evidences exploited by econometricians and professional forecasters (Forni et al., 2000, Giannone, Reichlin, and Small, 2008 and Stock and M. W. Watson, 2002, Stock and M. Watson, 2011 respectively) is that the dynamics

⁷The explanation of these estimation procedures will be provided in the next section while discussing the Dynamic Factor Model.

of macroeconomic time series are determined by a relative small number of latent, common factors plus uncorrelated disturbances, with the former being able to capture the state of the economy, the business cycle. Moreover, the use of latent factors has always been very popular in the asset pricing literature, from the CAPM (Sharpe, 1964), which could be seen as a one-factor model, to the Macroeconomic Approach of Chen, Roll, and Ross (1986). The main idea is the same: Financial markets are also driven by few factors which reproduce the business cycle.

Choosing a dynamic factor model does not only mean imposing a subjective perspective on how macroeconomic and financial variables evolve over time (based on macroeconomic fluctuations for instance), rather it also solves an important issue present in econometrics and machine learning, called the curse of dimensionality. A dynamic factor model is able to exploit all the pieces of information embedded in large panels of N -time series by reducing their dimensions to R latent components which explain most of the volatility of the series. In fact, the problem is actually overturned, from the curse to the blessing of dimensionality. Following the findings of Stock and M. W. Watson (2016), we are applying Principal Component Analysis to reduce the panel of financial variables and GDP growth rates described in the previous section and, consequently, to retrieve the unobserved latent factors.

However, the dynamic factor model represents only the first step when computing the Growth-at-Risk. More precisely, given the stochastic process $\{Y_t^i\}_{t \in \mathbb{R}^+}$ for the i -country GDP growth rate:

$$Y_{it+1} = \mu_{it+1|t} + \sqrt{\sigma_{it+1|t}^2} Z_{it+1} \quad \text{with: } Z_{it+1} \sim \text{iid } \mathcal{D}_{Z_i}(0; 1) \quad (20)$$

we estimate the conditional mean of the process $\mu_{it|t-1}$ using the DFM, while we are going to design the conditional variance $\sigma_{it|t-1}^2$ fitting different GARCH-type models (Bollerslev, 1986, Nelson, 1991, Zakoian, 1994) without specifying any distribution for the residuals Z_{it} in such a way to estimate non-parametrically the five percent quantile of the GDP conditional distributions. This is more than an estimation procedure, as we are imposing a precise *weltanschauung* consisting of the idea that financial variables are key determinants for the distribution of the GDP growth rates of european countries, which operate through the conditional mean of the processes. This structure will be back-tested both in-sample and out-of-sample in order to understand if national financial conditions are relevant predictors for the α -quantile density regions. Brownlees and Souza (2021) showed that the methodology applied by the IMF based on a quantile regression with financial variables as regressors is outperformed by a simple AR(1)-GARCH(1;1); we try to understand whether the predictor's shortfall of significance was due to adverse model selection or due to their lack of downside-risk predicting power.

In the dynamic factor model we have:

$$\begin{aligned} F_t &= \Psi_0 + \Psi_1 F_{t-1} + \dots + \Psi_p F_{t-p} + \eta_t \\ X_t &= \Lambda F_t + \varepsilon_t \end{aligned} \tag{21}$$

where the N -dimensional vector of GDP growth rates for seventeen European OECD countries plus the USA, could be decomposed into the R -dimensional vector of latent factors F_t and the N -vectors of idiosyncratic disturbances. The matrix Λ is the $(N \times R)$ matrix of factor loadings, with $N > R$. Moreover, the first equation describes the dynamics of the factors, which follows a Vector Autoregressive Process of order p .

The Dynamic Factor Model illustrated above represents the workhorse model for forecasting and nowcasting macroeconomic time series (Bańbura, Giannone, et al., 2013) and its estimation usually involves the use of the Kalman Filter and Smoother together with the expectation maximization algorithm (Doz, Giannone, and Reichlin, 2011, Bańbura and Modugno, 2014, Barigozzi and Luciani, 2019 among others). However, by imposing that ε_t has no cross-sectional dependence, id est a diagonal variance/covariance matrix, plus a set of assumptions on the residuals of the model, we are able to simplify the computation and to estimate what is called in the literature an Exact Dynamic Factor Model:

1. $E[\varepsilon_{jt} \varepsilon_{it}] = 0 \quad \forall j \neq i$
2. $E[\eta_t \eta_{t-k}] = 0 \quad \forall k > 0$
3. $E[\varepsilon_t \eta_t] = 0$
4. $\varepsilon_t = \delta(L)\varepsilon_{t-1} + \xi_t \quad \xi_t \sim \text{iid } N(0, \sigma^2)$

For the case in which we allow for mild cross-sectional dependence, thus a covariance matrix which is not necessarily diagonal, we have a Generalized Dynamic Factor Model. Our specification imposes a restrictive assumption on the vector of disturbances, but this simplifies the estimation procedure and reduces the computational power required, allowing for the implementation of a mixed frequency approach. Moreover, the exact formulation enables the estimation of the latent factors even for a relatively small panel of time series. First, the latent factors are estimated by the Principal Component Analysis, which allows us to identify jointly $\hat{\Lambda}\hat{F}_t$ up to the choice of a rotation matrix. We know that the following estimation procedure will give us a consistent estimate:

$$\lim_{N, T \rightarrow \infty} \hat{\Lambda}\hat{F}_t \xrightarrow{p} \Lambda F_t$$

Secondly, we fit a reduced form VAR of order p for the factors and a reduced form VAR of order q for the PCA idiosyncratic residuals $\hat{\varepsilon}_t = (X_t - \hat{\Lambda}\hat{F}_t)$:

$$\varepsilon_t = \delta_0 + \delta_1\varepsilon_{t-1} + \dots + \delta_q\varepsilon_{t-q} + \xi_t$$

lastly the optimal forecast of X_t under the quadratic loss function is obtained as follows⁸:

$$\begin{aligned} E[X_{t+1} | X_t F_{t+1}, \dots, X_{t-\infty} F_{t-\infty}] &= E[\Lambda F_{t+1} + \varepsilon_{t+1} | \mathcal{F}_t] \\ E[X_{t+1} | \mathcal{F}_t] &= (\hat{\Lambda}\hat{\Psi}(L) - \hat{\delta}(L)\hat{\Lambda})\hat{F}_t + \hat{\delta}(L)X_t \\ E[X_{t+1} | \mathcal{F}_t] &= \hat{\alpha}(L)\hat{F}_t + \hat{\delta}(L)X_t \end{aligned} \quad (23)$$

The detailed estimation of the reduced form VAR matrices $\delta(L)$ and $\Psi(L)$ are reported in appendix A. Following this procedure we are able to model the conditional mean of the process as:

$$\mu_{t+1|t} = E[X_{t+1} | \mathcal{F}_t]$$

which is needed for the estimation of the Growth-at-Risk. Equation (23) provides a closed form solution, both for the conditional mean and the one step ahead optimal forecast given the information set \mathcal{F}_t . Further, we derive the h -step ahead optimal forecasts by

⁸The proof is straightforward and follows from the exact dynamic factor model conditions:

$$\begin{aligned} E[X_{t+1} | X_t F_t, \dots, X_{t-\infty} F_{t-\infty}] &= E[\Lambda F_{t+1} + \varepsilon_{t+1} | \mathcal{F}_t] \\ E[X_{t+1} | \mathcal{F}_t] &= E[\Lambda\Psi(L)F_t | \mathcal{F}_t] + E[\delta(L)\varepsilon_t + \xi_{t+1} | \mathcal{F}_t] \\ E[X_{t+1} | \mathcal{F}_t] &= \hat{\Lambda}\hat{\Psi}(L)\hat{F}_t + E[\eta_{t+1} | \mathcal{F}_t] + E[\delta(L)\varepsilon_t | \mathcal{F}_t] + E[\xi_{t+1} | \mathcal{F}_t] \end{aligned}$$

The filtration \mathcal{F}_t could be expressed equivalently in terms of $\{X_t F_t, \dots, X_{t-\infty} F_{t-\infty}\}$ or $\{\eta_t \xi_t, \dots, \eta_{t-\infty} \xi_{t-\infty}\}$. Then, by conditioning η_{t+1} to all the past shocks and by exploiting the property of the conditional expectation:

$$E[X | Y] = \frac{Cov(X; Y)}{Var(X)} \quad (22)$$

we decompose the conditional expectation of $E[\xi_t | \mathcal{F}_t]$ as follow:

$$E[X_{t+1} | \mathcal{F}_t] = \hat{\Lambda}\hat{\Psi}(L)\hat{F}_t + E[\eta_{t+1} | \eta_t \xi_t, \eta_{t-1} \xi_{t-1}, \dots] + \hat{\delta}(L)\hat{\varepsilon}_t + E[\xi_{t+1}] + \frac{Cov(\xi_{t+1}; \eta_t \xi_t)}{Var(\xi_{t+1})}$$

then, under the normality assumption of ξ_t and η_t the conditional expectation can be dropped and the covariance is zero because the shocks are uncorrelated:

$$E[X_{t+1} | \mathcal{F}_t] = (\hat{\Lambda}\hat{\Psi}(L) - \hat{\delta}(L)\hat{\Lambda})\hat{F}_t + \hat{\delta}(L)X_t = \hat{\alpha}(L)\hat{F}_t + \hat{\delta}(L)X_t$$

recursion:

$$\begin{aligned}
E[X_{t+2} | \mathcal{F}_t] &= \hat{\alpha}(L)F_{t+1} + \hat{\delta}(L) E[X_{t+1} | \mathcal{F}_t] \\
E[X_{t+2} | \mathcal{F}_t] &= \hat{\alpha}(L)\Psi(L)F_t + \hat{\delta}(L) E[X_{t+1} | \mathcal{F}_t] \\
E[X_{t+3} | \mathcal{F}_t] &= \hat{\alpha}(L)\hat{\Psi}(L)^2F_t + \hat{\delta}(L) E[X_{t+2} | \mathcal{F}_t] \\
&\vdots \\
E[X_{t+h} | \mathcal{F}_t] &= \hat{\alpha}(L)\hat{\Psi}(L)^{h-1}\hat{F}_t + \hat{\delta}(L) E[X_{t+h-1} | \mathcal{F}_t]
\end{aligned} \tag{24}$$

It is important to notice that the optimal h -step ahead forecast could be seen as the linear combination of the forecast for the latent factors and the optimal forecast of the previous period. For the out-of-sample exercise we will build a block-bootstrap procedure which highlights the importance of also forecasting the future realizations of the vector of factors.

As previously mentioned, the second step consists of estimating the conditional variance $\sigma_{t+1|t}^2$ of the GDP growth rates of each countries. We begin with the GARCH(1;1) model that we estimate through the use of the Quasi Maximum Likelihood Estimator (QMLE), relying on the fundamental results of Lumsdaine (1996) about the asymptotic normality of the generalized autoregressive conditionally heteroskedastic model:

$$\begin{aligned}
\sigma_{it+1|t}^2 &= \omega_i + \alpha_i \kappa_{it}^2 + \beta_i \sigma_{it|t-1}^2 \\
&\text{where:}
\end{aligned} \tag{25}$$

$$\kappa_{it} = Y_{it} - \mu_{it|t-1} \quad \text{and} \quad \alpha_i + \beta_i < 1 ; \alpha_i > 0 \beta_i \geq 0 \omega_i > 0$$

even without bounding the fourth moment of the process. The vector of parameters $\theta_i = | \omega_i \alpha_i \beta_i |'$ is consistently estimated by maximizing the Gaussian quasi-likelihood $\mathcal{L}(\theta ; \kappa_1, \dots, \kappa_T)$:

$$\hat{\theta}_i = \arg \max_{\theta \in \Theta} \mathcal{L}(\theta ; \kappa_1, \dots, \kappa_T) = \arg \max_{\theta \in \Theta} \prod_{t=1}^T \frac{1}{\sqrt{2\pi\sigma_{it|t-1}^2}} \exp\left\{ -\frac{\kappa_{it}^2}{2\sigma_{it|t-1}^2} \right\} \tag{26}$$

even if the finite sample distribution of the GDP growth rate Y_{it} is not Gaussian. Further, we also model the possible presence of asymmetric effects inside the conditional variance dynamics by fitting a TAR(1;1) model for each country:

$$\begin{aligned}
\sigma_{it+1|t}^2 &= \omega_i + \alpha_i \kappa_{it}^2 + \gamma_i \kappa_{it}^2 \mathbf{1}_{\{\kappa_{it} < 0\}} + \beta_i \sigma_{it|t-1}^2 \\
&\text{with:} \\
\alpha_i + \gamma_i/2 + \beta_i &< 1 ; \alpha_i > 0 \beta_i > 0 \omega_i > 0 .
\end{aligned} \tag{27}$$

The estimation procedure is based on the QML estimator illustrated in equation (26), but

accounting for different dynamics of the conditional variance $\sigma_{it|t-1}^2$.

Unfortunately, the Quasi Maximum Likelihood Estimator produces poor results in the presence of few observations and the scenario worsens if we consider that for macroeconomic time series a larger dataset is required to obtain stable estimates of the GARCH-type model parameters. We confront this problem by also implementing the Composite Likelihood estimation proposed by Pakel, Shephard, and Sheppard (2011) and used by Brownlees and Souza (2021) for the GARCH(1;1) model. This procedure exploits the cross-sectional dimension of the dataset to compute the parameters of interest simultaneously instead of individually. Imposing equal conditional variance processes for the population parameters and across different countries might seem restrictive but the composite likelihood has the advantage of being able to solve the aforementioned instability problem and to deal with the presence of structural breaks. The reason why we implement both procedures is straightforward: as soon as the sample size increases (for example in the in-sample backtesting exercise or in the out-of-sample forecast using a recursive window) the reliability of the QMLE increases and comparing both estimation methods could be interesting.

As far as it concerns the CL estimation, sometimes also called Panel GARCH estimation procedure, we impose the persistence parameters α and β to be common across all the countries, while the nuisance parameters $\{\omega_1, \dots, \omega_N\}$ will be different. The GARCH specification is:

$$\sigma_{it+1|t}^2 = \omega_i + \alpha \kappa_{it}^2 + \beta \sigma_{it|t-1}^2$$

with:

(28)

$$\alpha + \beta < 1 ; \alpha > 0 \beta \geq 0 \omega_i > 0$$

The vector of parameters $\theta = | \alpha \beta \omega_1 \dots \omega_N |$ is estimated from the composite likelihood $\psi(\theta ; \kappa_{11}, \dots, \kappa_{1T}, \kappa_{21}, \dots, \kappa_{NT})$:

$$\hat{\theta} = \underset{\theta \in \Theta}{arg \max} \psi(\theta ; \mathcal{F}_t) = \underset{\theta \in \Theta}{arg \max} \prod_{t=1}^T \left(\prod_{i=1}^N \frac{1}{\sqrt{2\pi\sigma_{it|t-1}^2}} \exp\left\{ -\frac{\kappa_{it}^2}{2\sigma_{it|t-1}^2} \right\} \right)^{\frac{1}{N}} \quad (29)$$

which could be stated also in terms of the log-likelihood:

$$\hat{\theta} = \underset{\theta \in \Theta}{arg \max} \sum_{t=1}^T \left(\frac{1}{N} \sum_{i=1}^N -\frac{1}{2} \log(2\pi) - \frac{1}{2} \log(\sigma_{it|t-1}^2) - \frac{1}{2} \frac{\kappa_{it}^2}{\sigma_{it|t-1}^2} \right) \quad (30)$$

Noticable in equation (30), the composite log likelihood is nothing more than the maximum likelihood of the average of the probability density functions evaluated at each time horizon.

The only part missing in our dissertation about the Dynamic Factor-GARCH Model

is the explanation of the block-bootstrap algorithm that we created for the estimation of the h -step ahead GaR forecast but, for the clarity of exposition, we are going to illustrate it in the section dedicated to the out-of-sample exercise.

3.4 Backtesting Growth-at-Risk

In this section we are going to introduce the tests that we conducted for ranking the accuracy of the GaR models described before. However, before starting, we decided to investigate whether the choice of fitting a TARCH(1;1) specification is reasonable or not. For that reason we implement the negative sign and the negative size bias tests of Engle and Ng (1993):

- Negative size bias test: $\mathbf{z}_{it}^2 = \beta_0 + \beta_1 \mathbf{1}_{\mathbf{z}_{it-1} < 0} + \eta_t$
- Negative sign bias test: $\mathbf{z}_{it}^2 = \beta_0 + \beta_1 \mathbf{z}_{t-1} \mathbf{1}_{\mathbf{z}_{it-1} < 0} + \eta_t$

with $\mathbf{z}_{it}^2 = (Y_{it} - \mu_{it|t-1})^2 / \sigma_{it|t-1}^2$. For both tests we will compute the t -statistic associated with the $\hat{\beta}_1$ coefficients, under the null of no asymmetric effects between positive and negative GDP growth rates ($H_0 : \hat{\beta}_1 = 0$).

As far as it concerns the in-sample backtesting exercise, we are going to use three different criteria to rank all the Growth-at-Risk models for each european country; the first one is based on the number of "hitting". Given the empirical coverage metric, id est the empirical percentage of violation of the Growth-at-Risk occurred along the entire sample:

$$\hat{\pi}_i = \frac{1}{T} \sum_{t=1}^T \mathbf{1}_{\{Y_{it} < GaR_{it|t-1}^\alpha\}}$$

since the GaR is a α -conditional quantile, if the distribution of the GDP growth rate is correctly specified the number of hitting, or violation, should be equal to the confidence level α . For this reason, after computing $\hat{\pi}_i$, we will conduct the unconditional coverage test, under the null hypothesis that the empirical coverage is not statistically different from α . The test is based on the following statistic:

$$-2 \log \left(\frac{\alpha^{n1} (1 - \alpha)^{n0}}{\hat{\pi}^{n1} (1 - \hat{\pi})^{n0}} \right) \sim \chi_1^2 \quad (31)$$

where $n1$ is the number of violation and $n0$ the number of non-violation.

As a second step we will perform the dynamic quantile test of Engle and Manganelli (2004), under the assumption that the Growth-at-Risk is correctly specified, the sequence of violations

$$H_{it} = \begin{cases} 1 & \text{if } Y_{it} < GaR_{it|t-1}^\alpha \\ 0 & \text{otherwise} \end{cases} \quad (32)$$

is *iid*, meaning that we will not be able to predict future violations given the past realization of H_{it} . After estimating

$$W_i = \beta X_i + u_{it}$$

$$\text{with: } X_i = \begin{bmatrix} 1 & H_{iT-1} \\ \vdots & \vdots \\ 1 & H_{i1} \end{bmatrix} \quad \text{and} \quad W_{it} = \begin{bmatrix} H_T - \alpha \\ \vdots \\ H_{i2} - \alpha \end{bmatrix} \quad (33)$$

we construct the test statistic as follow:

$$DQ_i = \frac{\hat{\beta}' X_i' X_i \hat{\beta}}{\alpha(1 - \alpha)} \sim \chi_q^2 \quad (34)$$

where q is the dimension of the row vector β , which in this case is equal to 2. Under the null hypothesis H_0 the sequence of violations is *iid*, while the alternative hypothesis states the presence of some degree of dependence in H_t , meaning that the violations are going to cluster.

The last metric we are evaluating is the tick loss function (Giacomini and Komunjer, 2005, Giacomini and Komunjer, 2005 and Taylor, 2019), defined as:

$$TL_i = \frac{1}{T} \sum_{t=1}^T (\alpha - H_{it}) (Y_{it} - GaR_{it|t-1}^\alpha) \quad (35)$$

which puts an extra penalization of size $(1 - \alpha)$ on the violation of the Growth-at-Risk, while in the other case the weight on the deviation of the GDP growth rate from the conditional quantile is equal to α . To conclude, in this section we have introduced the criteria which will be used to backtest the explanatory power of the GaR models. While all the metrics presented so far will be implemented also in the out-of-sample exercise, in this second analysis we will make use of another important hypothesis test, the Diebold-Mariano Equal Predictive Ability Test (Diebold and Mariano, 2002 and Diebold, 2015), in order to directly compare the Growth-at-Risk forecast and to rank the models in terms of tick loss minimization.

3.5 Out-of-Sample Growth-at-Risk Forecast

The second step of our empirical analysis consists of an out-of-sample forecast competition among the Growth-at-Risk models. What might seem like a simple forecasting horse race actually embodies the main part of our paper, trying to understand the impact of financial variables on the European GDP growth rates and their predicting power. As explained at the end of section 3.3, we are going to apply all the tests introduced before

to our forecast estimates plus the Diebold-Mariano Equal Predictive Ability Test. More precisely, we are going to test the predicting power of different models under the tick loss function (equation 35) for different time lengths of the forecast.

Given i -different models and the aforementioned tick loss function:

$$\mathcal{L}_{it+h} = \text{TL}(Y_{t+h} - \widehat{GaR}_{it+h|t}^\alpha) \quad (36)$$

we are going to construct the loss differential d_t defined as:

$$d_\tau = \Delta \mathcal{L}_t = \mathcal{L}_{i\tau} - \mathcal{L}_{j\tau} \quad \text{with: } i \neq j \quad \text{for } \tau = t+1, \dots, t+H. \quad (37)$$

The DM test statistic is constructed as follow:

$$DM = \frac{\bar{d}_t}{\sqrt{\text{Var}(\bar{d}_t)/H}} \quad (38)$$

However, since the loss differential exhibits problems of serial correlation, the long-run variance of its mean will be computed using the Newey-West Heteroschedasticity and Autocorrelation Consistent (HAC) estimator (Newey and West, 1986), based on a non-parametric kernel estimator:

$$\begin{aligned} \text{Var}(\bar{d}_t) &= \left(H^{-1} \sum_{\tau=t+1}^{t+H} d_\tau d'_\tau \right)^{-1} \widehat{S} \left(H^{-1} \sum_{\tau=t+1}^{t+H} d_\tau d'_\tau \right)^{-1} \\ \widehat{S} &= \left(H^{-1} \sum_{\tau=t+1}^{t+H} d_\tau \right) = \sum_{j=-q+1}^{q+1} \left(1 - \frac{|j|}{q} \right) \widehat{\Gamma}_j \end{aligned} \quad (39)$$

where q is known as the bandwidth parameter, $(q-1)$ is the lag length, $\widehat{\Gamma}_j$ is the sample autocorrelation at horizon j . For the choice of q we rely on the rule-of-thumb provided by Newey and West (1994) which consists of $q(H) = 4(H/100)^{\frac{2}{9}}$. For $H \rightarrow \infty$ the DM test statistic converges in distribution to a standard normal, allowing for the use of the z_α -score, then we conducted a one-side Diebold Mariano test:

$$\begin{cases} H_0 : \bar{d}_t = 0 \\ H_A : \bar{d}_t > 0 \end{cases} \quad (40)$$

Formally the null hypothesis tells us that the two models have the same predictive power since $\bar{d}_t \xrightarrow{p} E[d_t] = 0$, while under the alternative $E[d_t] > 0$, id est the model j has a lower tick loss function and, consequently, a higher predictive power vis-a-vis model i .

Before analysing the results of our estimations we will provide the explanation of

how we construct the h -step ahead prediction regions. While producing the one step ahead GaR is straightforward and it only requires data until the forecast origin, for larger horizons the quantile forecasts could be produced only relying on simulation procedures. Regarding the AR(1) class of models, we implemented the procedure developed by Brownlees and Souza (2021), on the other hand, for the DFM, we built a block bootstrap type algorithm (Lahiri, 1999) which keeps track of the possible presence of dependence in the time series by resampling from the matrix of the fitted residuals $\{\widehat{Z}_{it-H}, \dots, \widehat{Z}_{i1}\}_{i=1}^N$.⁹ In both procedures, the optimal block dimension corresponds to the length of the forecast, meaning that for the one step ahead the algorithms rely on standard resampling procedures. While the implementation for the AR(1) is straightforward, the Dynamic Factor Model requires two extra steps, first, retrieving the realization of the latent factors by PCA decomposition, and secondly, forecasting their future realization $\{\widehat{F}_{t+1|t}, \dots, \widehat{F}_{t+H-1|t}\}$ for the estimation of the conditional mean of the process.¹⁰

To conclude, the pseudo-codes of the algorithms are illustrated in the next pages; it is important to mention that in the out-of-sample exercise we use a recursive window procedure with the re-calibration of the models at each time step. The bootstrap procedures will be used also for the estimation of the Panel GARCH, the only difference being that while the country specific parameters will be continuously re-estimated, the persistence parameters (α and β) will be fixed after the first estimation.

⁹For the sake of clarity, both algorithms are visualized at the end of the current section.

¹⁰Up to $t + H - 1$ because Y_{t+H} depends on the previous realization of the factors.

Algorithm 1 h -step ahead GaR forecast for the AR(1)-GARCH Model

INPUTS

- (a) Y : ($T \times N$) matrix of GDP growth rates.
- (b) H : forecast horizon.
- (c) B : number of bootstrap replications.

PROCEDURE

1. Estimate the AR(1) models and their conditional mean.
2. Estimate the GARCH model for the conditional variance.
3. Construct the matrix \widehat{Z}_t of the residuals.
4. Construct the vector S by randomly drawing from the uniform distribution $\{1, \dots, T - h\}$
5. For i in $\{1, \dots, N\}$ do

For b in $\{1, \dots, B\}$ do

For h in $\{1, \dots, H\}$ do

If $h=1$ do

$$\mu_{it+1|t} = \hat{\phi}_{i0} + \hat{\phi}_{i1}Y_{it}$$

$$\sigma_{it+1|t}^2 = \omega_i + \alpha_i(Y_{it} - \mu_{it|t-1})^2 + \beta_i\sigma_{it|t-1}^2$$

$$\widetilde{Y}_{it+1|t}^b = \mu_{it+1|t} + \sqrt{\sigma_{it+1|t}^2} \widehat{Z}_{ib}$$

If $h \in [2; H]$ do

$$\mu_{it+h|t+h-1}^b = \hat{\phi}_{i0} + \hat{\phi}_{i1}\widetilde{Y}_{it+h-i|t+h-2}^b$$

$$\sigma_{it+h|t+h-1}^{2;b} = \omega_i + \alpha_i(\widetilde{Y}_{it+h-1}^b - \mu_{it+h-1|t+h-2}^b)^2 + \beta_i\sigma_{it+h-1|t+h-2}^{2;b}$$

$$\widetilde{Y}_{it+h|t+h-1}^b = \mu_{it+h|t+h-1}^b + \sqrt{\sigma_{it+h|t+h-1}^{2;b}} \widehat{Z}_{ib+h}$$

6. From the matrix \widetilde{Y}_{t+h}^b of dimension ($B \times N$), for i in $\{1, \dots, N\}$ do

$$\text{GaR}_{it+h|t}^\alpha = \widehat{F}_{\widetilde{Y}_{t+h}^b}^{-1}(\alpha)$$

OUTPUT

- (a) $\text{GaR}_{t+h|t}^\alpha$: ($1 \times N$) vector of h -step ahead GaR forecast.
-
-

Algorithm 2 h -step ahead GaR forecast for the DFM-GARCH Model

INPUTS

- (a) Y_t : ($T \times N$) matrix of GDP growth rates.
- (b) Y_{pca} : ($T \times K$) matrix for the PCA, last N columns are the GDP growth rates.
- (c) r : number of latent factors.
- (d) p : number of lags for the VAR of factors.
- (e) q : number of lags for the VAR of residuals.
- (f) H : forecast horizon.
- (g) B : number of bootstrap replications.

PROCEDURE

1. Estimate the latent factors by PCA decomposition of Y_{pca} .
2. Estimate the DFM following the mixed frequency approach.
3. Estimate the GARCH models for the conditional variances.
4. Construct the matrix \hat{Z}_t of the residuals.
5. Construct the vector S by randomly drawing from the uniform distribution $\{1, \dots, T - h\}$
6. Generate the Forecast $\{\hat{F}_{t+1}, \dots, \hat{F}_{t+H-1}\}$
7. For h in $\{1, \dots, H\}$ do

For b in $\{1, \dots, B\}$ do

If $h=1$ do

$$\begin{aligned} \mu_{t+1|t} &= \hat{\alpha}(L)F_t + \hat{\delta}(L)Y_t \\ \sigma_{it+1|t}^2 &= \omega_i + \alpha_i(Y_{it} - \mu_{it|t-1})^2 + \beta_i\sigma_{it|t-1}^2 \\ \tilde{Y}_{it+1|t}^b &= \mu_{it+1|t} + \sqrt{\sigma_{it+1|t}^2} \hat{Z}_{ib} \end{aligned}$$

If $h \in [2; H]$ do

$$\begin{aligned} \mu_{t+h|t+h-1}^b &= \hat{\alpha}(L)\hat{F}_{t+h-1} + \hat{\delta}(L) \text{E}[\tilde{Y}_{t+h-1}^b | \mathcal{F}_{t-h-2}^b] \\ \sigma_{it+h|t+h-1}^{2;b} &= \omega_i + \alpha_i(\tilde{Y}_{it+h-1}^b - \mu_{it+h-1|t+h-2}^b)^2 + \beta_i\sigma_{it+h-1|t+h-2}^{2;b} \\ \tilde{Y}_{it+h|t+h-1}^b &= \mu_{it+h|t+h-1}^b + \sqrt{\sigma_{it+h|t+h-1}^{2;b}} \hat{Z}_{ib+h} \end{aligned}$$

8. From the matrix \tilde{Y}_{t+h}^b of dimension $(B \times N)$, for i in $\{1, \dots, N\}$ do

$$\text{GaR}_{it+h|t}^\alpha = \hat{F}_{\tilde{Y}_{it+h}^b}^{-1}(\alpha)$$

OUTPUT

- (a) $\text{GaR}_{t+h|t}^\alpha$: ($1 \times N$) vector of h -step ahead GaR forecast.
-
-

4 Empirical Analysis

Now that we have elaborated our methodology and theoretical framework, we will present our findings in this fourth section. Hereby, distinguishing between the in-sample backtesting exercise, to assess and rank the accuracy of the different models, and the out-of-sample forecasting competition. We present the parameters obtained from fitting the variations of ours and Brownlees and Souza (2021)'s univariate model to our data - for the quantile regression approach we report the coefficients for each forecast horizon respectively. In fact, for the non-parametric approach established by Brownlees and Souza (2021), we choose to model the conditional variance with a GARCH, TARCh, Factor-GARCH and Panel-GARCH specification. Further, we will comment on the results we obtain from running the tests specified in the previous section.

4.1 Backtesting

In line with our exposition of the previous section, we implement the negative sign and the negative size bias test as a first step, to assess the tenability of the TARCh(1,1) specification implemented in our Dynamic Factor Model and the univariate approach based on an autoregressive process of order one (reported as "AR1"). The results (table 1) suggest the absence of asymmetric effects for most of the countries in our sample¹¹, only Greece being an outlier by exhibiting significant indication of sign bias for both models, as well as indication of size bias in case of the AR(1) approach. Overall, we decide to disregard the asymmetric TARCh specification for the out-of-sample analysis due to the lack of significance.¹²

Table 1: Sign/Size Bias Test

Test		AUT	ISL	GBR	DNK	ESP	NOR	FIN	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC	
DFM	Size	t-stat	0.0891	1.2773	1.5997	0.2009	1.7713	0.4002	0.3346	-0.5771	-1.0357	-1.9794	-1.0847	0.8200	-1.1604	-1.0439	1.4258	0.7788	-0.3102	1.2371
		p-value	0.9291	0.2032	0.1115	0.8410	0.0783	0.6895	0.7384	0.5646	0.3018	0.0494	0.2796	0.4134	0.2475	0.2980	0.1558	0.4372	0.7568	0.2177
	Sign	t-stat	0.4229	-1.7111	-1.9560	0.0163	-0.1081	-0.5209	-0.7484	0.3451	0.7730	0.3691	0.2247	-0.0211	1.6050	0.7260	-0.2659	-0.8138	0.5223	-2.1812
		p-value	0.6729	0.0889	0.0521	0.9870	0.9141	0.6031	0.4553	0.7304	0.4406	0.7125	0.8225	0.9832	0.1104	0.4688	0.7907	0.4169	0.6021	0.0305
AR 1	Size	t-stat	-0.0548	0.2256	1.7781	0.5257	0.6682	-0.3872	2.1084	0.8914	0.2690	0.4297	0.8658	0.0540	1.1046	0.8058	1.4234	0.5826	-0.4306	2.4273
		p-value	0.9563	0.8218	0.0772	0.5998	0.5049	0.6991	0.0365	0.3740	0.7883	0.6680	0.3878	0.9570	0.2709	0.4215	0.1565	0.5610	0.6673	0.0163
	Sign	t-stat	0.1290	0.0849	-0.5249	-1.3731	-0.7399	0.3507	-1.0397	0.1713	1.1384	-0.9214	-0.0625	-0.0596	-0.0041	-0.0525	-0.9781	-0.1938	0.2681	-2.3036
		p-value	0.8975	0.9324	0.6003	0.1715	0.4604	0.7262	0.3000	0.8642	0.2566	0.3582	0.9502	0.9525	0.9967	0.9582	0.3294	0.8465	0.7889	0.0225

¹¹DEU and FIN exhibiting some indication of size bias, for the DFM and the AR(1) approach respectively.

¹²However, we will present our estimation and test results for the TARCh(1,1) specification in the in-sample backtesting section, will refrain from commenting on it though.

4.1.1 Parameters

The complete estimation results for our Dynamic Factor Model (DFM) and the non-parametric approach (AR1) of Brownlees and Souza (2021) are displayed in table 8. We report the parameters for each specification of the models across all countries in our sample. The DFM specifications for the conditional variance comprise of the GARCH, TARCH and Panel-GARCH approach; for the AR(1) approach we explore GARCH, TARCH, Factor-GARCH and Panel-GARCH specifications. The GARCH estimation results show that the majority of countries exhibit persistent volatility dynamics for both models - only Switzerland, Germany and Finland not reaching convergence in estimation and therefore not displaying any GARCH effects at all. As mentioned in the previous section, for estimating the GARCH-type specifications of our models we rely on the Quasi Maximum Likelihood Estimator, which is subject to poorer results when presented with scarce data. Moreover, the estimation of GARCH-model parameters for macro time series is reliant on very big data sets to create stable results. Given the relatively small size of our sample and the low frequency (quarterly) we impose, removing a single observation (country) from our model will change the results drastically. We solve this problem by also modeling a Panel-GARCH (or Composite Likelihood) specification for our DFM and the univariate model, which is able to exploit cross-sectional commonality in GARCH dynamics. By aggregating the time series of all countries in our sample the Composite Likelihood is able to provide us with precise and representative parameter estimates for the GARCH(1,1). For the DFM we obtain very strong persistence with $\alpha = 0.1489$ and $\beta = 0.8511$; also the univariate AR(1) model exhibits persistent GARCH effects, with the CL persistence being 0.9912 ($\alpha = 0.3114, \beta = 0.6798$).

In case of the quantile regression approach we report our estimation results in table 9. The table reports the estimated coefficient together with the respective t-statistic and p-value for each forecasting horizon $h = 1, \dots, 4$ and each predictor¹³. Importantly, the applied approach can be essentially seen as a regression where the coefficients express the marginal effects of the variables on the specified quantiles, therefore the coefficients obtained are to be interpreted differently than the ones we obtained from the other two models. Here, the absolute value of the coefficients is key, as it directly represents the weight of the respective variable, or predictors in this case. Our findings are in line with the ones of Brownlees and Souza, 2021; The NFCI displays the highest relevance as down side risk predictor for the majority of countries in our sample and throughout all forecasting horizons, while gradually losing predictive power in the long run. In fact, all variables turn out to be not statistically significant at all time horizons bigger than one.

¹³Predictors used: GDP growth rate, NFCI, TS and HP

4.1.2 Tests

Similarly to Brownlees and Souza (2021), our backtesting approach is based on the classic methodology developed in the Risk Management literature. The tests conducted in this section are: (1) the assessment of empirical coverage, capturing the percentage of GaR violations; (2) the unconditional coverage test, assessing whether the empirical percentage of hitting is different from the α -confidence level chosen a priori for the quantile; (3) the Dynamic quantile test, testing if the sequence of GaR violations are independent and identically distributed, by assessing if past violations can predict future ones; (4) the value of the tick loss, which puts an extra weight on the violation of the conditional quantile. Additionally, we conduct a Ljung-Box Correlation Test on the residuals and squared residuals: the Quasi-Maximum Likelihood only estimates the parameters consistently if the conditional mean and the conditional variance of the process are correctly specified, meaning that the sequence of standardized residuals $\{Z_{it}\}_{t \in \mathbb{R}^+}$ should not show any dependence in absolute and squared levels. Once again, when we choose the Normal distribution for the maximum likelihood estimation of GARCH-type models we must ensure the correct specification of the conditional mean not only to obtain consistent parameter estimates, but also to validate our estimation procedure and the one of Brownlees and Souza (2021).

In tables 10 & 11 we report the complete results of all the tests applied for the in-sample analysis for each country respectively, while tables 2 & 3 summarise our findings in a more concise manner.

Table 2: Hypothesis testing: percentage of rejections, in sample analysis.

	Model	Unconditional Coverage Test	Dynamic Quantile Test	Ljung-Box Test Residuals	Ljung-Box Test Residuals ²
DFM	GARCH	11.11	5.56	5.56	5.56
	TARCH	5.56	11.11	11.11	5.56
	CL-GARCH	5.56	5.56	5.56	0.00
AR1	GARCH	0.00	33.33	61.11	0.00
	TARCH	0.00	16.67	61.11	0.00
	CL-GARCH	5.56	16.67	50.00	0.00
	F-GARCH	0.00	11.11	38.89	0.00
QR		0.00	16.67	-	-

Table 2 reports the percentage of countries for which the null of the respective test conducted has been rejected, for all three of the models analysed in this paper. The quantile regression performs very well in terms of unconditional coverage, with none of the 18 countries analysed in our sample exhibiting empirical coverage significantly different

from the 95%-level, but exhibiting some increased degree of clustered violations that indicate dependence in the sequence of hitting of the quantile regression. While the AR(1) approach seems to outperform the DFM in terms of unconditional coverage¹⁴, it is the opposite case for the Dynamic Quantile test; the DFM outperforms the AR(1) approach in every specification, indicating less clustering in violations and a more correct specification of the model. These results, however, should be considered under the light of the Correlation Tests of the residuals and the squared residuals for the above mentioned reasons. We obtain significantly better results for the DFM approach when analysing the residuals; for the squared residuals, the DFM exhibits some kind of correlation in roughly 5% of the cases¹⁵.

Table 3: Empirical Coverage (in %) and Tick Loss function, in-sample analysis.

	Model	Coverage	Tick-Loss
DFM	GARCH	4.66 [2.75 5.93]	0.07 [0.05 0.11]
	TARCH	4.24 [3.39 5.93]	0.07 [0.05 0.11]
	CL-GARCH	5.08 [4.24 5.93]	0.08 [0.05 0.11]
AR1	GARCH	4.24 [3.39 7.2]	0.10 [0.06 0.14]
	TARCH	4.24 [2.75 5.72]	0.09 [0.06 0.13]
	CL-GARCH	5.08 [4.45 6.78]	0.10 [0.07 0.15]
	F-GARCH	4.24 [3.39 5.72]	0.09 [0.05 0.13]
QR		5.08 [3.81 5.93]	0.09 [0.06 0.11]

Table 3 reports the median and the respective quartiles across all countries of our sample, for both the empirical coverage and the tick loss. Even if the unconditional coverage test was rejected for some specification of the Dynamic Factor Model, the empirical percentage of violations are close to 5%. Remarkably, the AR(1)-GARCH(1,1) displays the widest interquantile range and the lowest empirical coverage together with the DFM-TARCH(1,1) and the AR(1)-TARCH(1,1). Regarding the tick loss, our results show that the DFM outperforms the other two models in every specification. Figures 4 - 8 show, that our model is not only able to keep track of the realizations of the GDP growth

¹⁴CL-GARCH specification exhibiting the same amount of rejection

¹⁵One country only - In this case, Portugal

rates of each country, but also the dynamics of the conditional distributions, and also the Growth-at-Risk conditional quantiles follow the business cycle fluctuations. Moreover, the fifth quantiles produced by the AR(1) models turn out to be smoother and further away from the GDP growth rates - also ones produced by the quantile regression exhibit similarly smooth behaviour.

To conclude, our empirical findings highlight how our model outperforms the other two in-sample, exploiting the predictive power of the financial variables. In the next section we are going to conduct a forecasting exercise to understand the out-of-sample predicting power of the models.

4.2 Out-of-sample analysis

In the following we present the results of our out of sample analysis, conducted from the one up to the four step ahead forecast horizon. As previously mentioned, we do not use the TARCH(1,1) specification given the rejection arising from the negative size and negative sign bias tests (table 1). Moreover, we used 45% of the data set for validation of the out-of-sample exercise, using the first quarter of 1997 as forecast origin. For clarity, all country specific results are listed in the Tables section (tables 12 - 13), while the results at aggregate level are reported in this section (tables 4 - 5).

The first important evidence to be mentioned is that the results are very heterogeneous considering different countries and different forecast horizons. In particular, the Dynamic Factor-GARCH Model increases the quality of its performance considering longer forecast horizons for all the econometric measures considered; on the contrary, the quantile regression performs poorly at all forecast horizons, with the only exception of the tick loss functions. Further, another important finding disregards the panel-GARCH estimation procedure, which does not show any ability of improving the forecast accuracy, both considering the DFM and the AR(1). From a computational perspective, we did not observe any improvement in terms of stability of the parameters; in fact, we decided to fix the parameters (α β) at the beginning of the forecasting exercise, as the algorithm very often failed to reach convergence in the maximization of the composite likelihood.

In table 4 we report the median and 25-75% interquantile range of the Empirical Coverage (in percent) and tick loss, for each forecast horizon. In terms of empirical coverage, the AR(1)-GARCH(1,1) model of Brownlees and Souza (2021) produces the best GaR forecasts, which are also less dispersed from the median values. As previously mentioned, the DFM increases its performance at longer horizons, both in terms of empirical coverage and tick loss. In fact, our model exhibits tick loss which is smaller or at least equal than the ones of the AR(1) class of models for $h \in [2; 4]$, even if the empirical coverage is always smaller than the theoretical confidence level of 5%. Lastly, the quantile regression

Table 4: Empirical Coverage (%) and Tick Loss, out-of-sample analysis.

		h = 1		h = 2		h = 3		h = 4	
	Model	Coverage	Tick-Loss	Coverage	Tick-Loss	Coverage	Tick-Loss	Coverage	Tick-Loss
DFM	GARCH	10.13 [7.59 11.39]	0.12 [0.08 0.18]	2.56 [1.28 4.81]	0.11 [0.09 0.17]	3.9 [2.6 4.87]	0.12 [0.09 0.18]	3.95 [2.96 5.26]	0.12 [0.11 0.2]
	CL-GARCH	8.86 [6.33 11.39]	0.11 [0.07 0.16]	2.56 [1.6 4.81]	0.12 [0.09 0.18]	3.9 [2.6 6.17]	0.12 [0.09 0.17]	3.95 [2.63 6.58]	0.12 [0.11 0.19]
AR1	GARCH	5.06 [4.11 7.28]	0.09 [0.07 0.15]	3.85 [3.85 5.13]	0.12 [0.09 0.17]	5.19 [3.9 6.17]	0.12 [0.1 0.17]	5.26 [3.95 6.58]	0.12 [0.1 0.16]
	CL-GARCH	6.33 [5.06 7.59]	0.09 [0.08 0.17]	5.13 [3.85 6.09]	0.11 [0.1 0.17]	5.19 [3.9 6.17]	0.12 [0.1 0.17]	5.26 [3.95 7.57]	0.12 [0.1 0.18]
QR		10.13 [5.38 13.61]	0.09 [0.08 0.16]	11.54 [7.69 14.1]	0.11 [0.08 0.16]	12.99 [5.84 16.88]	0.12 [0.09 0.16]	13.16 [9.21 17.11]	0.13 [0.1 0.18]

approach of Adrian, Boyarchenko, and Giannone (2016) is farther away from the true confidence level, with a median coverage being always above 10%.

In table 5 we display the percentage of rejection of the unconditional coverage and the dynamic quantile tests: For both tests the null hypotheses represent the correct specification of the Growth-at-Risk forecasts. The DFM and the AR(1) models present

Table 5: Unconditional Coverage and Dynamic Quantile Tests, percentage of rejection, out-of-sample analysis

		h = 1		h = 2		h = 3		h = 4	
	Model	UC	DQ	UC	DQ	UC	DQ	UC	DQ
DFM	GARCH	38.89	66.67	16.67	33.33	5.56	55.56	0.00	66.67
	CL-GARCH	33.33	72.22	22.22	55.56	5.56	66.67	0.00	72.22
AR1	GARCH	5.56	33.33	5.56	61.11	0.00	66.67	0.00	61.11
	CL-GARCH	0.00	61.11	0.00	72.22	5.56	72.22	5.56	72.22
QR		38.89	61.11	55.56	77.78	61.11	77.78	55.56	88.89

UC: Unconditional Coverage test; DQ: Dynamic Quantile test

almost the same performance, only during the one step ahead time horizon the AR(1)-GARCH(1,1) turns out to be more accurate. The quantile regression always shows higher percentages of rejection for both tests. Generally speaking, none of the models are able to produce iid sequences, with the smallest percentage of rejection being equal to 33%.

The last part of our out-of-sample validation concerns the Superior Predictive Ability Test of Diebold and Mariano (2002). Table 16 report the results as follows: Each coefficient represents the percentage of times that the model on the row has outperformed the model in the column by implementing the test at country level; our panel of interest is the left one. Obviously, the results are very heterogeneous and we do not observe evidence about the presence of a model which is systematically outperforming the others. How-

ever, even for this hypothesis test our model increases its performance at longer forecast horizons.

To conclude, we do not find strong evidence about an increasing predicting power of the Dynamic Factor-GARCH Model with respect to the model of Brownlees and Souza (2021), while the quantile regression turned out to have the lowest level of overall accuracy for the Growth-at-Risk forecasts. The most important evidence arising from our out-of-sample analysis concerns the high degree of heterogeneity of the results: while for some countries the financial conditions increase the goodness of the forecast, for others a better accuracy could be obtained by simply using an AR(1)-GARCH(1,1). Moreover, we also proved that by modelling the conditional mean of the process $\{Y_{it}\}_{t \in \mathbb{R}}$ using a dynamic factor model, whose principal components are estimated from a panel of financial conditions, leads to better GaR forecast compared to a quantile regression that relies on the same financial variables as downside risk predictors. Lastly, the increased performance of our model at longer time horizons, might suggest that the latent factors extrapolated from the panel of financial conditions have more predicting power in the long run.

In the next section we are going to focus our attention on a smaller out-of-sample period to understand if the financial variables increase their predicting power, by considering a smaller sample around the great recession.

4.3 A Focus on the Great Recession

As an additional analysis we focus our attention on a smaller out-of-sample forecasting period. To be precise, we are going to validate the models only on 35% of our data set, starting from the second quarter of 2001 as forecast origin. The reason for this second exercise is to understand whether the financial variables increase their importance as downside risk predictors and therefore also their predictive power around the period of the Great Recession, or not - in case they do, we want to test whether the models in question are able to capture this important effect.

Table 6 shows that the Dynamic Factor-GARCH Model increases its performance both in terms of tick loss and empirical coverage; in fact, it shows the smallest tick loss at each forecast horizons considered even if the empirical coverage is a bit smaller than the theoretical confidence level for $h \in [2; 4]$. Our model is the only one which improves its performance in this second forecast evaluation, while the other models do not exhibit any enhancement. In particular, the quantile regression has the same tick loss as in the previous analysis and the empirical coverage is always above ten percent, meaning that this estimation procedure is not able to capture the increased predicting power of financial condition as our model is doing.

Regarding the Unconditional Coverage and the Dynamic Quantile tests, table 7 shows

Table 6: Empirical Coverage and Tick Loss, percentage of rejection, out-of-sample analysis around the great recession.

		h = 1		h = 2		h = 3		h = 4	
Model		Coverage	Tick-Loss	Coverage	Tick-Loss	Coverage	Tick-Loss	Coverage	Tick-Loss
DFM	GARCH	8.06 [5.24 11.29]	0.07 [0.05 0.17]	3.28 [1.64 4.51]	0.10 [0.07 0.18]	3.33 [3.33 6.67]	0.12 [0.08 0.16]	5.08 [3.39 5.08]	0.12 [0.1 0.18]
	CL-GARCH	8.06 [5.24 11.29]	0.08 [0.04 0.17]	3.28 [2.05 6.15]	0.12 [0.06 0.17]	3.33 [3.33 6.67]	0.14 [0.08 0.16]	5.08 [3.39 6.36]	0.12 [0.09 0.18]
AR1	GARCH	5.65 [4.84 6.45]	0.10 [0.06 0.24]	6.56 [4.92 6.56]	0.11 [0.08 0.18]	6.67 [5 6.67]	0.12 [0.08 0.19]	5.93 [5.08 8.05]	0.13 [0.09 0.19]
	CL-GARCH	6.45 [4.84 8.06]	0.10 [0.06 0.24]	6.56 [4.92 6.56]	0.11 [0.06 0.17]	6.67 [5 6.67]	0.12 [0.07 0.18]	6.78 [5.51 6.78]	0.13 [0.09 0.18]
QR		12.1 [4.84 15.73]	0.09 [0.06 0.19]	13.93 [8.61 17.62]	0.11 [0.06 0.16]	13.33 [7.5 20.83]	0.12 [0.07 0.18]	14.41 [11.86 21.61]	0.14 [0.08 0.19]

that our model decreases the percentage of rejection at each time horizon compared to the previous out-of-sample analysis. Contrasting, the other two models decrease their performance at almost all the forecast horizons considered. Once again, this opposite behavior of the Dynamic Factor Model and the quantile regression confirms that the approach of Adrian, Boyarchenko, and Giannone (2016) is not able to fully exploit the predicting power of financial conditions.

Table 7: Unconditional Coverage and Dynamic Quantile Tests, share of rejection for the period around the great recession (in%)

		h = 1		h = 2		h = 3		h = 4	
Model		UC	DQ	UC	DQ	UC	DQ	UC	DQ
DFM	GARCH	33.33	55.56	22.22	33.33	0.00	50.00	0.00	72.22
	CL-GARCH	33.33	50.00	33.33	38.89	5.56	55.56	0.00	72.22
AR1	GARCH	11.11	38.89	16.67	61.11	11.11	66.67	16.67	66.67
	CL-GARCH	5.56	22.22	5.56	55.56	5.56	66.67	11.11	66.67
QR		61.11	61.11	61.11	77.78	66.67	88.89	77.78	88.89

Lastly, in the right panel of figure 16 we provide the results for the Diebold-Mariano test: Also in this case our model increases the percentage of times in which it outperforms the AR(1) and the quantile regression, while the other two specification do not display any improvement. We can conclude that financial variables increase their importance in terms of predicting the GDP downside risk, but only the Dynamic Factor-GARCH Model is able to exploit this predictive power; in fact, both the AR(1) and the quantile regression worsened the quality of their GaR forecasts.

5 Conclusion

Are financial conditions relevant predictors for European GDP growth rate downside risk? This was the main question we dedicated our paper to: We tried to provide an answer using a data-driven approach for modelling the Growth-at-Risk. More precisely, we propose a Dynamic Factor-GARCH model which computes the conditional distribution of the GDP growth rates non-parametrically, exploiting the dimensions of a panel of national financial conditions. Moreover, we also develop a blockbootstrap algorithm to produce the GaR forecasts at longer time horizons numerically, in order to compare the performance of our model with the ones of Adrian, Boyarchenko, and Giannone (2016) and Brownlees and Souza (2021) out-of-sample.

We conduct an in-sample backtesting analysis, as developed in the Risk Management literature and find, that our model outperforms the other two, both in terms of producing iid sequences of hitting and tick loss. Moreover, our approach is able to produce the best specification for the standardized residuals, displaying independence in absolute and squared levels. Contrasting to our in-sample results, the out-of sample results exhibit a higher degree of heterogeneity across countries. While our model performs at least as good or better as the AR(1)-GARCH(1,1) specification of Brownlees and Souza (2021) in the long run, it produces unsatisfactory results for the one-step forecast horizon. However, by focusing our out-of-sample analysis on a smaller sample around the period of the Great Recession, we not only outperform the other two models analysed, but also obtain strong indication of increased importance and predicting power of financial conditions. In fact, only our Dynamic Factor Model manages to exploit the aforementioned dynamics, while the AR(1) and quantile regression approach actually even decrease the quality of their GaR forecasts.

We provide evidence that by correctly modelling financial conditions, they not only exhibit predictive ability for GDP downside risk, but also improve in-sample GaR predictions. Further, we show that they are relevant out-of-sample predictors in the long run. Finally, when focusing on periods of extraordinary financial distress, like the Great Recession, financial conditions become even more relevant. However, the right model needs to be applied in order to exploit that predicting power.

Concluding, we find that the relationship between finance and macroeconomic outcomes becomes stronger in light of the European integration. We hope our paper could inspire practitioners to further push this important ongoing debate.

Table 9: Quantile Regression Parameters

h	Coef.	AUT	ISL	GBR	DNK	ESP	NOR	FIN	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC	
1	Intep _{t-1}	value	-0.6681	-1.4253	-0.5991	-1.0658	-0.6938	-1.3631	-1.6010	-1.0195	-0.8591	-0.9181	-0.8967	-0.8694	-0.9723	-1.1237	-0.6162	-1.1009	-0.6086	-3.7262
		t-stat	-3.5565	-2.9437	-5.2404	-5.6858	-2.7049	-3.8236	-4.8743	-2.8201	-4.7182	-3.7507	-2.6783	-4.4444	-4.9388	-1.5003	-3.6014	-2.2115	-2.8341	-5.6335
		p-value	0.0005	0.0037	0.0000	0.0000	0.0075	0.0002	0.0000	0.0054	0.0000	0.0002	0.0081	0.0000	0.0000	0.1354	0.0004	0.0283	0.0052	0.0000
	GDP _{t-1}	value	-0.0542	-0.5347	0.1505	-0.1268	0.2631	-0.1809	-0.1267	0.4863	0.0185	0.2122	-0.0967	-0.0373	0.5130	0.0385	0.4527	-0.5456	0.2233	0.3351
		t-stat	-0.1732	-2.3130	0.7894	-0.9870	1.4589	-1.1878	-0.7037	2.3061	0.1054	0.8249	-0.5581	-0.2013	4.9764	0.1214	1.8542	-1.6511	1.9592	1.5257
		p-value	0.8627	0.0219	0.4310	0.3251	0.1464	0.2366	0.4826	0.0223	0.9162	0.4106	0.5775	0.8407	0.0000	0.9036	0.0654	0.1006	0.0517	0.1290
	NFCI _{t-1}	value	-0.4406	-0.5638	-0.1874	-0.4302	-0.1065	-0.2027	-0.9426	-0.0171	-0.2620	-0.0875	-0.6084	-0.4710	-0.0810	0.3206	-0.1251	-0.2995	-0.0604	-1.1463
		t-stat	-1.2852	-1.1932	-3.7649	-2.5212	-3.2002	-1.2607	-3.4567	-0.4291	-3.0827	-0.8437	-3.3432	-2.2054	-2.6721	0.7003	-1.9696	-3.2628	-3.0490	-2.5691
		p-value	0.2005	0.2345	0.0002	0.0126	0.0016	0.2091	0.0007	0.6684	0.0024	0.4000	0.0010	0.0288	0.0083	0.4847	0.0505	0.0033	0.0027	0.0111
TS _{t-1}	value	0.1114	0.2652	0.2144	0.0509	0.1698	-0.1017	0.2053	0.2132	0.3315	-0.0268	-0.2833	-0.0394	0.1397	0.1220	0.1828	0.5777	0.2639	0.0559	
	t-stat	0.6579	1.6556	4.4270	0.5379	1.6881	-0.8679	1.4398	3.0820	3.9654	-0.1794	-0.8888	-0.6465	3.0950	0.5838	1.7191	3.1356	3.3828	1.0093	
	p-value	0.5115	0.0996	0.0000	0.5913	0.0932	0.3867	0.1518	0.0024	0.0001	0.8579	0.3754	0.5188	0.0023	0.5601	0.0874	0.0020	0.0009	0.3143	
HP _{t-1}	value	0.0973	0.8317	0.0417	-0.0518	-0.0693	0.1131	0.1373	0.0093	0.1101	-0.0559	0.0791	-0.0738	-0.0509	0.5374	-0.0514	0.0951	0.0262	0.0989	
	t-stat	0.9571	2.6906	0.9613	-1.4685	-0.6341	1.5633	1.4024	0.1616	2.8783	-0.3384	0.2289	-0.6421	-0.8049	1.4046	-1.0362	0.4524	0.8575	0.4494	
	p-value	0.3399	0.0078	0.3378	0.1438	0.5268	0.1198	0.1626	0.8718	0.0045	0.7355	0.8192	0.5217	0.4220	0.1620	0.3016	0.6516	0.3924	0.6537	
2	Intep _{t-2}	value	-0.8460	-2.7308	-0.5615	-1.2211	-0.8652	-1.3965	-1.4410	-0.6329	-0.8125	-1.0757	-1.2202	-1.3477	-1.4531	-1.7331	-0.6599	-2.1711	-0.8341	-3.9795
		t-stat	-2.5880	-3.6528	-3.0464	-4.6606	-3.0199	-4.0798	-6.9483	-3.4266	-2.9391	-1.1251	-3.1041	-5.5679	-4.1416	-2.2657	-4.9985	-3.2628	-3.3761	-7.5674
		p-value	0.0105	0.0003	0.0027	0.0000	0.0029	0.0001	0.0000	0.0008	0.0038	0.2621	0.0022	0.0000	0.0001	0.0247	0.0000	0.0013	0.0009	0.0000
	GDP _{t-2}	value	0.1050	0.4959	0.0953	-0.2131	0.7833	-0.0630	0.2554	0.2064	0.2435	-0.0140	0.1313	0.4873	0.4805	0.1769	-0.0042	0.0162	0.3299	-0.1834
		t-stat	0.3676	2.9407	0.9583	-1.6882	7.2174	-0.3455	1.7185	1.5396	1.6724	-0.0342	1.0289	3.1679	2.5808	0.4283	-0.0177	0.0730	1.8034	-0.9437
		p-value	0.7136	0.0037	0.3393	0.0932	0.0000	0.7301	0.0875	0.1255	0.0963	0.9728	0.3050	0.0018	0.0107	0.6690	0.9859	0.9419	0.0731	0.3467
	NFCI _{t-2}	value	-0.5502	0.8012	-0.2601	-0.3665	-0.0037	0.1240	-0.5368	-0.1133	-0.1575	-0.1079	-0.3147	-0.4078	0.0294	0.5545	-0.1072	-0.0261	-0.0412	-1.5990
		t-stat	-1.8328	1.4242	-4.5441	-2.1625	-0.1914	0.6108	-1.7514	-3.2532	-1.1010	-0.7923	-1.6415	-2.1376	0.4374	1.2181	-1.6190	-0.2567	-1.5187	-2.8288
		p-value	0.0686	0.1562	0.0000	0.0320	0.8485	0.5421	0.0812	0.0014	0.2725	0.4293	0.1026	0.0340	0.6624	0.2249	0.1073	0.7977	0.1307	0.0052
TS _{t-2}	value	0.1296	0.5573	0.2245	0.1602	0.1054	0.1395	0.1482	0.1722	0.2543	0.0589	-0.0001	0.0126	0.2122	0.1853	0.1716	0.8260	0.3327	-0.0133	
	t-stat	1.0506	2.5458	3.7405	1.3991	1.2789	0.6491	1.9780	3.4617	1.0267	-0.0005	0.2198	2.3776	1.3464	1.9830	3.1698	4.6648	-0.2039		
	p-value	0.2949	0.0118	0.0003	0.1636	0.2027	0.5171	0.0496	0.0023	0.0007	0.8993	0.9996	0.8263	0.0185	0.1800	0.0490	0.0018	0.0000	0.8387	
HP _{t-2}	value	0.0052	0.7461	-0.0162	0.0015	-0.0608	-0.0054	0.0234	0.0586	0.0257	0.0577	0.7447	-0.1533	-0.0645	0.6280	-0.0060	0.3026	0.0507	0.3272	
	t-stat	0.0573	1.0210	-0.3264	0.0757	-1.1103	-0.0828	0.3702	1.8720	0.6439	0.1700	7.3492	-1.3764	-0.6440	1.2965	-0.1234	1.8538	1.1844	1.4262	
	p-value	0.9544	0.3087	0.7445	0.9397	0.2684	0.9341	0.7117	0.0629	0.5205	0.8652	0.0000	0.1705	0.5204	0.1966	0.9020	0.0655	0.2379	0.1557	
3	Intep _{t-3}	value	-0.9223	-1.5056	-0.4761	-1.4663	-0.9760	-1.3592	-1.5555	-0.7337	-0.6484	-0.9156	-1.2518	-1.3186	-1.4356	-1.4378	-0.6566	-2.3936	-0.6497	-3.9118
		t-stat	-2.5342	-2.1623	-2.8454	-5.5183	-2.1468	-4.2749	-4.5545	-2.7429	-2.9126	-1.2912	-2.3541	-5.1586	-3.3676	-3.8209	-3.3996	-3.2397	-2.1414	-7.9373
		p-value	0.0122	0.0320	0.0050	0.0000	0.0332	0.0000	0.0000	0.0068	0.0041	0.1984	0.0197	0.0000	0.0009	0.0002	0.0008	0.0014	0.0337	0.0000
	GDP _{t-3}	value	0.0660	0.0306	0.0731	0.1555	0.3759	-0.0842	0.5067	0.1324	-0.1712	-0.1584	0.0494	0.4223	0.3481	0.1145	-0.3699	0.3980	0.0856	-0.1994
		t-stat	0.1775	0.1022	0.5517	1.1160	1.5133	-0.5016	4.3641	0.8894	-1.2839	-0.3901	0.2241	1.8941	1.3327	4.6864	-1.5458	1.3842	0.5443	-1.4888
		p-value	0.8593	0.9187	0.5819	0.2660	0.1321	0.6166	0.0000	0.3751	0.2009	0.6970	0.8229	0.0599	0.1844	0.0000	0.1240	0.1681	0.5869	0.1384
	NFCI _{t-3}	value	-0.4099	0.2486	-0.2480	-0.4026	0.0125	0.0895	-0.1739	-0.0868	0.0225	-0.0048	-0.3101	-0.5378	-0.0017	0.3433	-0.1368	0.0722	-0.0280	-1.5293
		t-stat	-1.5358	0.3902	-4.7818	-2.1025	0.3752	0.3526	-0.9941	-4.8217	0.3211	-0.0419	-1.0781	-2.3177	-0.0203	1.1720	-3.7486	0.6864	-1.3260	-3.8732
		p-value	0.1265	0.6969	0.0000	0.0370	0.7080	0.7248	0.3216	0.0000	0.7485	0.9666	0.2825	0.0217	0.9839	0.2428	0.0002	0.4934	0.1867	0.0002
TS _{t-3}	value	0.2746	0.2647	0.1651	0.0067	0.0600	0.1295	0.1549	0.2154	0.2565	0.2044	-0.0792	-0.0163	0.1509	0.0761	0.2178	0.5742	0.3184	0.0396	
	t-stat	1.5468	0.9449	2.1664	0.0454	0.4944	0.7364	1.5700	2.3463	3.3142	0.7822	-0.2040	-0.2375	1.5476	0.5925	2.7753	1.8309	2.4840	0.7166	
	p-value	0.1238	0.3461	0.0317	0.9638	0.6216	0.4625	0.1183	0.0201	0.0011	0.4352	0.8386	0.8126	0.1236	0.5543	0.0061	0.0689	0.0140	0.4746	
HP _{t-3}	value	0.0059	0.6709	-0.1878	0.0126	-0.0041	0.0308	-0.0366	0.0917	0.0912	-0.1039	0.2516	-0.0318	0.1114	0.6020	0.0380	0.0348	0.0227	0.1579	
	t-stat	0.0581	1.1507	-2.4443	0.2654	-0.0335	0.4736	-0.5052	1.6654	2.7479	-0.4747	1.2706	-0.2078	0.7712	2.5265	0.7976	0.1409	0.5228	0.7250	
	p-value	0.9538	0.2515	0.0155	0.7910	0.9733	0.6364	0.6141	0.0977	0.0067	0.6356	0.2056	0.8356	0.4417	0.0124	0.4262	0.8881	0.6018	0.4694	
4	Intep _{t-4}	value	-0.8951	-1.6284	-0.3738	-1.3725	-0.9917	-1.3918	-1.4435	-0.8693	-0.7646	-0.8595	-1.3527	-1.2871	-0.8082	-1.3804	-0.6997	-1.8317	-0.4123	-4.4103
		t-stat	-6.4083	-2.5101	-1.9587	-4.7794	-3.0537	-4.5590	-4.8067	-2.7757	-2.1943	-0.7656	-1.8388	-3.8859	-1.7772	-4.2968	-3.0528	-3.4234	-1.2645	-9.4797
		p-value	0.0000	0.0130	0.0518	0.0000	0.0026	0.0000	0.0000	0.0061	0.0296	0.4450	0.0677	0.0001	0.0774	0.0000	0.0026	0.0008	0.2078	0.0000
	GDP _{t-4}	value	-0.1609	-0.1034	-0.2256	0.1197	0.3561	-0.1048	0.1491	0.1416	-0.3751	0.1413	-0.0117	0.0219	-0.1413	-0.2140	-0.4533	0.3368	-0.0417	0.5121
		t-stat	-1.4714	-0.3408	-4.7043	0.8622	3.7481	-0.7039	1.3710	0.9446	-2.2342	0.3405	-0.0434	0.1145	-0.6656	-0.9454	-1.5969	1.2288	-0.2237	3.0041
		p-value	0.1431	0.7337	0.0000	0.3898	0.0002	0.4825	0.1722	0.3462	0.0268	0.7339	0.9654	0.9090	0.5066	0.3458	0.1122	0.2209	0.8233	0.0031
	NFCI _{t-4}	value	-0.2611	0.5552	-0.1080	-0.3107														

Table 10: Country specific hypothesis testing: in sample analysis

Coverage

Model		AUT	ISL	GBR	DNK	ESP	NOR	FIN	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC
DFM	GARCH	2.5424	7.6271	3.3898	5.0847	2.5424	9.3220	5.9322	0.8475	5.9322	4.2373	12.7119	3.3898	5.0847	2.5424	3.3898	5.0847	2.5424	5.9322
	TARCH	1.6949	5.9322	3.3898	5.0847	3.3898	9.3220	5.9322	2.5424	5.9322	5.0847	12.7119	3.3898	6.7797	3.3898	3.3898	3.3898	2.5424	5.0847
	CL-GARCH	3.3898	5.0847	7.6271	5.0847	2.5424	7.6271	5.9322	4.2373	5.9322	4.2373	9.3220	4.2373	9.3220	1.6949	4.2373	5.0847	4.2373	5.9322
AR1	GARCH	3.3898	8.4746	2.5424	3.3898	4.2373	7.6271	7.6271	4.2373	4.2373	4.2373	7.6271	5.9322	5.0847	1.6949	3.3898	2.5424	7.6271	5.9322
	TARCH	4.2373	8.4746	1.6949	5.0847	1.6949	8.4746	5.0847	4.2373	4.2373	3.3898	7.6271	5.9322	3.3898	2.5424	3.3898	2.5424	6.7797	2.5424
	CL-GARCH	3.3898	9.3220	4.2373	5.0847	2.5424	10.1695	7.6271	5.0847	5.0847	5.0847	6.7797	6.7797	6.7797	3.3898	6.7797	3.3898	7.6271	5.0847
	F-GARCH	4.2373	9.3220	5.9322	2.5424	2.5424	4.2373	4.2373	5.9322	4.2373	3.3898	6.7797	2.5424	3.3898	4.2373	5.9322	2.5424	3.3898	5.0847
QR		5.0847	7.6271	3.3898	5.9322	5.0847	5.9322	5.0847	3.3898	6.7797	3.3898	6.7797	6.7797	5.9322	5.0847	5.0847	3.3898	5.9322	2.5424

Unconditional Coverage Test

Model		AUT	ISL	GBR	DNK	ESP	NOR	FIN	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC		
DFM	GARCH	t-stat 1.8163 p-value 0.1777	1.4874 0.2226	0.7228 0.3952	0.0018 0.9664	1.8163 0.1777	3.7404 0.0531	0.2042 0.6513	6.4612 0.0110	0.2042 0.6513	0.1521 0.6966	10.5525 0.0012	0.7228 0.3952	0.2042 0.6513	0.7228 0.3952	0.7228 0.3952	0.0018 0.9664	0.7228 0.3952	0.7228 0.6513	0.2042 0.6513	
	TARCH	t-stat 3.6069 p-value 0.0575	0.2042 0.6513	0.7228 0.3952	0.0018 0.9664	0.7228 0.3952	3.7404 0.0531	0.2042 0.6513	1.8163 0.1777	0.2042 0.6513	0.0018 0.9664	10.5525 0.0012	0.7228 0.3952	0.7228 0.3952	0.7228 0.3952	0.7228 0.3952	0.7228 0.3952	0.0018 0.9664	0.7228 0.3952	0.0018 0.9664	
	CL-GARCH	t-stat 0.7228 p-value 0.3952	0.0018 0.9664	1.4874 0.2226	0.0018 0.9664	1.8163 0.1777	1.4874 0.2226	0.2042 0.6513	0.1521 0.6966	0.2042 0.6513	0.1521 0.6966	3.7404 0.0531	0.1521 0.6966	5.1770 0.0229	3.6069 0.0575	0.1521 0.6966	0.0018 0.9664	0.0018 0.9664	0.1521 0.6966	0.1521 0.6966	0.2042 0.6513
AR 1	GARCH	t-stat 0.7228 p-value 0.3952	2.5045 0.1135	1.8163 0.1777	0.7228 0.3952	0.1521 0.6966	1.4874 0.2226	1.4874 0.2226	0.1521 0.6966	0.1521 0.6966	0.1521 0.6966	1.4874 0.2226	0.2042 0.6513	0.0018 0.9664	3.6069 0.0575	0.7228 0.3952	1.8163 0.1777	1.4874 0.2226	1.8163 0.1777	0.2042 0.6513	
	TARCH	t-stat 0.1521 p-value 0.6966	2.5045 0.1135	3.6069 0.0575	0.0018 0.9664	3.6069 0.0575	2.5045 0.1135	0.0018 0.9664	0.1521 0.6966	0.1521 0.6966	0.7228 0.3952	1.4874 0.2226	0.2042 0.6513	0.1521 0.6966	0.1521 0.6966	0.1521 0.6966	1.8163 0.1777	0.7228 0.3952	1.8163 0.1777	0.7228 0.3952	0.0018 0.9664
	CL-GARCH	t-stat 0.7228 p-value 0.3952	3.7404 0.0531	0.1521 0.6966	0.0018 0.9664	1.8163 0.1777	5.1770 0.0229	1.4874 0.2226	0.0018 0.9664	0.0018 0.9664	0.0018 0.9664	0.7114 0.3990	0.7114 0.3990	0.7114 0.3990	0.7228 0.3952	0.7114 0.3990	0.7228 0.3952	1.4874 0.2226	1.4874 0.2226	0.0018 0.9664	
	F-GARCH	t-stat 0.1521 p-value 0.6966	3.7404 0.0531	0.2042 0.6513	1.8163 0.1777	1.8163 0.1777	0.1521 0.6966	0.1521 0.6966	0.2042 0.6513	0.1521 0.6966	0.7228 0.3952	0.7114 0.3990	1.8163 0.1777	0.7228 0.3952	0.1521 0.6966	0.2042 0.6513	1.8163 0.1777	0.7228 0.3952	0.7228 0.3952	0.0018 0.9664	
QR	t-stat 0.0018 p-value 0.9664	2.5045 0.1135	0.7228 0.3952	0.2042 0.6513	0.2042 0.6513	0.2042 0.6513	0.2042 0.6513	0.2042 0.6513	0.7228 0.3952	0.7114 0.3990	0.7228 0.3952	1.4874 0.2226	0.7114 0.3990	0.2042 0.6513	0.0018 0.9664	0.2042 0.6513	0.1521 0.6966	0.7114 0.3990	0.7228 0.3952		

Dynamic Quantile Test

Model		AUT	ISL	GBR	DNK	ESP	NOR	FIN	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC
DFM	GARCH	t-stat 3.3647 p-value 0.8493	6.7763 0.4525	5.8243 0.5604	2.9834 0.8865	1.6687 0.9759	5.6481 0.5814	2.9728 0.8875	6.2766 0.5078	13.5212 0.0604	4.7290 0.6930	25.5567 0.0006	3.3138 0.8545	4.7859 0.6861	1.8495 0.9677	1.0490 0.6030	5.4684 0.6030	2.1922 0.9484	12.1862 0.0946
	TARCH	t-stat 3.1986 p-value 0.8660	3.2118 0.8647	5.8359 0.5590	2.9262 0.8917	5.6338 0.5831	5.8735 0.5546	2.9728 0.8875	10.0858 0.1838	13.5184 0.0604	4.5268 0.7175	25.5795 0.0006	3.2303 0.8629	18.4570 0.0101	1.0433 0.9941	1.0490 0.9940	10.7488 0.1500	2.1922 0.9484	4.4551 0.7261
	CL-GARCH	t-stat 1.3337 p-value 0.9875	6.1101 0.5270	9.8963 0.1945	2.8593 0.8977	9.1528 0.2419	13.9399 0.0523	3.2942 0.8565	3.3271 0.8532	13.7218 0.0564	3.8007 0.8024	13.7575 0.0557	6.3628 0.4981	28.3607 0.0002	2.9961 0.8854	2.0985 0.9542	13.5846 0.0591	2.0239 0.9585	2.4418 0.9314
AR 1	GARCH	t-stat 5.3703 p-value 0.6149	15.9238 0.0258	9.3734 0.2269	5.3747 0.6143	7.8341 0.3475	3.8996 0.7913	15.4740 0.0304	14.4226 0.0442	3.8356 0.7985	12.9776 0.0727	22.2171 0.0000	31.6688 0.0000	4.2409 0.7516	2.7490 0.9072	1.1058 0.9929	8.4531 0.2943	14.1698 0.0482	11.2948 0.1263
	TARCH	t-stat 15.7793 p-value 0.0272	10.3352 0.1704	2.7794 0.9046	5.2436 0.6303	2.7221 0.9095	9.4895 0.2194	7.6941 0.3603	14.2027 0.0477	3.8216 0.8001	7.9754 0.3348	22.2104 0.0023	6.3235 0.5025	2.8639 0.8973	8.8416 0.2642	1.0952 0.9931	8.5045 0.2902	13.4592 0.0617	1.6335 0.9773
	CL-GARCH	t-stat 5.3668 p-value 0.6153	14.2972 0.0461	16.3976 0.0217	4.2831 0.7467	16.3148 0.0224	13.9810 0.0515	10.3097 0.1717	10.5435 0.1598	4.9613 0.6647	7.1403 0.4144	9.2907 0.2325	8.0593 0.3274	10.0415 0.1862	5.5840 0.5891	6.4574 0.4875	5.2402 0.6307	11.6259 0.1136	3.2412 0.8618
	F-GARCH	t-stat 3.7110 p-value 0.8124	8.1433 0.3201	12.9839 0.0725	1.6558 0.9764	1.6197 0.9779	2.2485 0.9448	17.0586 0.0170	4.7791 0.6869	3.9168 0.7893	8.4617 0.2936	35.7395 0.0000	7.1581 0.4126	1.6058 0.9784	4.3851 0.7345	3.6783 0.8160	3.2530 0.8607	1.4218 0.9849	2.8949 0.8946
QR	t-stat 6.7910 p-value 0.4510	5.0249 0.6569	6.8376 0.4460	11.3410 0.1244	17.1801 0.0163	1.3962 0.9857	6.0831 0.5301	6.4784 0.4851	4.5622 0.7132	8.2145 0.3141	18.0245 0.0119	33.8643 0.0000	11.9042 0.1038	4.5732 0.7119	2.4610 0.9300	8.1898 0.3162	4.7519 0.6902	1.7089 0.9742	

Tick Loss

Model		AUT	ISL	GBR	DNK	ESP	NOR	FIN	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC
DFM	GARCH	0.0460	0.1684	0.0614	0.0831	0.0427	0.1101	0.0922	0.0482	0.0479	0.0682	0.1505	0.0744	0.0441	0.1724	0.0544	0.0802	0.0310	0.1693
	TARCH	0.0456	0.1773	0.0619	0.0844	0.0426	0.1130	0.0922	0.0462	0.0479	0.0682	0.1510	0.0741	0.0445	0.1826	0.0544	0.0733	0.0310	0.1676
	CL-GARCH	0.0445	0.1746	0.0618	0.0878	0.0446	0.1133	0.0972	0.0462	0.0482	0.0722	0.1318	0.0782	0.0469	0.1764	0.0599	0.0886	0.0334	0.1697
AR1	GARCH	0.0610	0.2438	0.0632	0.1018	0.0621	0.1263	0.1437	0.0683	0.0609	0.0971	0.1653	0.0975	0.0557	0.1864	0.0583	0.1207	0.0457	0.2048
	TARCH	0.0613	0.2479	0.0613	0.1034	0.0602	0.1271	0.1267	0.0671	0.0612	0.0914	0.1647	0.0985	0.0559	0.2627	0.0589	0.1206	0.0469	0.1984
	CL-GARCH	0.0599	0.2367	0.0667	0.1070	0.0646	0.1430	0.1506	0.0672	0.0688	0.1087	0.1619	0.0950	0.0613	0.1794	0.0670	0.1222	0.0475	0.2030
	F-GARCH	0.0480	0.2041	0.0516	0.0960	0.0576	0.1285	0.1258	0.0484	0.0539	0.0883	0.1478	0.0820	0.0543	0.1913	0.0468	0.0978	0.0386	0.1890
QR		0.0564	0.1945	0.0567	0.0889	0.0699	0.1094	0.1061	0.0627	0.0570	0.0913	0.1613	0.0928	0.0638	0.1939	0.0587	0.0919	0.0424	0.1803

Table 11: Country specific hypothesis testing: Ljung-Box Correlation Test for residuals and the squared residuals, in sample analysis

Residuals

Model		AUT	ISL	GBR	DNK	ESP	NOR	FIN	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC		
DFM	GARCH	χ^2	13.6595	7.1585	15.1419	19.8320	11.7975	7.7045	11.0382	10.6238	16.9315	6.7897	14.1664	5.5843	9.6248	5.7428	4.8583	16.3852	11.6449	11.0486	
		p-value	0.1891	0.7104	0.1270	0.0309	0.2988	0.6577	0.3545	0.3876	0.0759	0.7451	0.1655	0.8489	0.4740	0.8364	0.9004	0.0891	0.3095	0.3537	
	TARCH	χ^2	12.7539	7.7455	14.8493	19.6950	11.7921	7.5422	11.0382	11.0458	16.6999	6.1742	14.2814	5.5640	8.7061	7.0963	4.8583	19.6077	11.6449	10.7379	
		p-value	0.2378	0.6537	0.1377	0.0323	0.2992	0.6735	0.3545	0.3540	0.0813	0.8004	0.1605	0.8505	0.5602	0.7163	0.9004	0.0332	0.3095	0.3783	
	CL-GARCH		χ^2	11.9237	9.3990	14.1734	17.3071	10.2563	8.0996	11.6595	10.0048	12.4753	5.3376	21.5839	4.9221	9.9158	5.3286	5.2105	18.0125	8.5173	8.4548
			p-value	0.2902	0.4947	0.1652	0.0678	0.4183	0.6191	0.3085	0.4401	0.2545	0.8675	0.0174	0.8963	0.4479	0.8682	0.8767	0.0548	0.5784	0.5845
AR 1	GARCH	χ^2	10.9184	33.1408	12.4261	25.4682	88.4965	16.0812	35.4188	19.7271	19.8062	15.0600	22.2104	27.1535	3.2009	22.4810	11.9037	18.9649	15.5468	57.5160	
		p-value	0.3639	0.0003	0.2576	0.0045	0.0000	0.0973	0.0001	0.0319	0.0311	0.1299	0.0141	0.0025	0.9763	0.0128	0.2916	0.0407	0.1134	0.0000	
	TARCH	χ^2	11.9940	32.6615	13.8531	25.0951	85.2452	14.8911	32.0963	21.1278	19.7228	14.8919	22.1870	27.0695	3.3468	41.5820	12.0151	18.4838	17.4995	59.3918	
		p-value	0.2855	0.0003	0.1798	0.0052	0.0000	0.1361	0.0004	0.0202	0.0320	0.1361	0.0142	0.0025	0.9720	0.0000	0.2840	0.0473	0.0640	0.0000	
	CL-GARCH		χ^2	11.5740	36.4507	11.0838	24.7531	92.0829	18.3335	16.3487	24.8904	11.8227	13.4588	27.7679	28.2008	4.8153	17.6362	7.9639	18.3551	12.3469	56.3087
			p-value	0.3146	0.0001	0.3510	0.0058	0.0000	0.0496	0.0901	0.0056	0.2971	0.1991	0.0020	0.0017	0.9032	0.0614	0.6324	0.0493	0.2625	0.0000
F-GARCH		χ^2	12.6676	17.0375	8.0216	27.6272	34.1046	6.9964	22.2657	15.2838	20.0764	7.8734	24.0257	18.3045	7.4658	16.5055	10.9643	20.9668	14.2960	41.4168	
		p-value	0.2429	0.0735	0.6267	0.0021	0.0002	0.7258	0.0138	0.1221	0.0285	0.6412	0.0075	0.0500	0.6809	0.0860	0.3603	0.0213	0.1599	0.0000	

Squared Residuals

Model		AUT	ISL	GBR	DNK	ESP	NOR	FIN	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC		
DFM	GARCH	χ^2	3.4260	9.3917	5.9558	9.4927	4.9659	2.7071	5.9885	11.4272	4.9766	6.2048	1.9957	19.1747	7.7007	1.2663	6.2685	7.8069	6.1661	15.8842	
		p-value	0.9696	0.4954	0.8190	0.4861	0.8934	0.9875	0.8162	0.3252	0.8927	0.7978	0.9964	0.0381	0.6580	0.9995	0.7922	0.6477	0.8011	0.1030	
	TARCH	χ^2	4.7713	8.6049	6.1986	9.4853	5.0265	2.6180	5.9885	10.0269	5.0242	6.1407	2.0473	19.1197	10.4923	1.7279	6.2685	10.7104	6.1661	14.6120	
		p-value	0.9059	0.5700	0.7983	0.4867	0.8894	0.9890	0.8162	0.4381	0.8896	0.8033	0.9960	0.0388	0.3984	0.9980	0.7922	0.3805	0.8011	0.1469	
	CL-GARCH		χ^2	5.4877	8.6709	2.5100	11.2401	3.9012	3.0336	6.5099	9.0901	5.9792	4.6702	2.3007	11.8918	6.2915	0.3896	5.6508	7.2539	4.5418	15.5762
			p-value	0.8563	0.5636	0.9907	0.3391	0.9517	0.9806	0.7708	0.5236	0.8170	0.9121	0.9935	0.2924	0.7902	1.0000	0.8437	0.7013	0.9196	0.1124
AR 1	GARCH	χ^2	13.0828	29.5993	17.0887	19.6780	26.9493	13.2198	7.5926	15.1796	10.0135	13.4191	13.3415	11.2583	6.2496	0.4418	5.0905	10.3108	5.4122	14.2194	
		p-value	0.8738	0.0766	0.6472	0.4782	0.1367	0.8678	0.9942	0.7660	0.9679	0.8587	0.8623	0.9392	0.9985	1.0000	0.9997	0.9622	0.9995	0.8192	
	TARCH	χ^2	13.9516	29.0521	23.9687	21.9815	29.1073	14.4715	9.6740	18.0746	9.8055	26.6435	13.1909	10.4268	6.4807	1.7514	6.7458	10.3710	11.4750	21.2169	
		p-value	0.8329	0.0867	0.2438	0.3415	0.0857	0.8058	0.9737	0.5825	0.9716	0.1456	0.8690	0.9598	0.9981	1.0000	0.9974	0.9609	0.9330	0.3845	
	CL-GARCH		χ^2	12.1471	29.7191	13.7185	20.6345	23.1542	14.7059	1.8655	19.8144	25.3492	12.4013	2.6349	7.7435	19.1641	0.3684	10.6881	18.1830	10.1697	18.1180
			p-value	0.9109	0.0745	0.8445	0.4189	0.2813	0.7930	1.0000	0.4696	0.1884	0.9016	1.0000	0.9934	0.5112	1.0000	0.9539	0.5754	0.9650	0.5796
F-GARCH		χ^2	6.0317	7.8795	9.2225	5.6769	5.7046	3.7155	10.4295	8.2637	7.0087	5.4470	4.2483	14.2325	5.6322	0.4656	4.5538	12.1274	5.5309	4.9933	
		p-value	0.8126	0.6406	0.5111	0.8416	0.8394	0.9593	0.4037	0.6031	0.7246	0.8594	0.9355	0.1626	0.8452	1.0000	0.9189	0.2766	0.8530	0.8916	

Table 12: Country specific hypothesis testing: one and two step ahead forecast horizons, out of sample analysis

Coverage																		
Model	Coverage																	
	AUT	ISL	GRR	DNK	ESP	NOR	FIN	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC
DFM	GARCH																	
	2.561	6.103	5.182	2.561	0.000	2.561	3.862	1.281	1.281	2.561	3.862	0.000	3.862	7.623	1.281	1.281	0.000	8.974
DFM	CL-GARCH																	
	2.561	12.335	3.862	2.561	0.000	1.281	1.281	2.561	2.561	3.862	0.000	3.862	11.385	2.561	1.281	1.281	0.000	10.254
ARI	GARCH																	
	5.182	2.561	5.182	3.862	3.862	8.974	3.862	3.862	5.182	3.862	7.623	3.862	6.103	0.000	3.862	5.182	3.862	8.974
ARI	CL-GARCH																	
	5.182	6.103	5.182	5.182	5.182	7.623	5.182	3.862	3.862	10.254	3.862	6.103	5.182	2.561	3.862	3.862	6.103	6.061
QR	7.623 12.335 12.335 8.974 11.385 11.385 11.385 11.385 11.385 11.385 11.385 11.385 11.385 11.385 11.385 11.385 11.385 11.385 11.385																	
Unconditional Coverage Test																		
Model	Unconditional Coverage Test																	
	AUT	ISL	GRR	DNK	ESP	NOR	FIN	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC
DFM	GARCH																	
	1.170	0.310	0.027	1.170	8.008	1.170	0.267	3.191	1.170	1.170	0.267	8.008	0.267	1.025	0.267	8.008	1.170	2.196
DFM	CL-GARCH																	
	1.170	7.187	0.267	1.170	8.008	1.170	0.027	3.191	1.170	1.170	0.267	8.008	0.267	3.218	1.170	0.027	3.191	3.396
ARI	GARCH																	
	0.027	1.170	0.027	0.267	2.196	0.267	0.267	0.267	0.267	0.267	0.267	0.267	0.267	0.267	0.267	0.267	0.267	0.267
ARI	CL-GARCH																	
	0.027	0.310	0.027	0.027	1.025	0.027	0.027	0.267	0.267	0.267	0.267	0.267	0.267	0.267	0.267	0.267	0.267	0.267
QR	1.025 0.310 0.027 0.027 0.267 0.267 0.267 0.267 0.267 0.267 0.267 0.267 0.267 0.267 0.267 0.267 0.267 0.267 0.267																	
Dynamic Quantile Test																		
Model	Dynamic Quantile Test																	
	AUT	ISL	GRR	DNK	ESP	NOR	FIN	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC
DFM	GARCH																	
	17.367	3.851	47.471	1.692	3.897	2.676	12.675	2.174	2.172	19.671	2.092	3.897	34.322	7.360	2.308	30.855	3.897	34.922
DFM	CL-GARCH																	
	17.414	39.499	31.311	3.970	3.897	2.765	20.626	2.276	3.083	18.516	3.079	3.897	34.322	28.108	17.460	39.749	3.970	69.499
ARI	GARCH																	
	46.985	2.067	48.114	35.904	35.271	16.393	9.798	20.152	24.887	9.720	2.267	9.435	88.2121	3.897	1.972	19.079	15.902	27.825
ARI	CL-GARCH																	
	46.985	2.067	48.114	35.904	35.271	16.393	9.798	20.152	24.887	9.720	2.267	9.435	88.2121	3.897	1.972	19.079	15.902	27.825
QR	46.985 2.067 48.114 35.904 35.271 16.393 9.798 20.152 24.887 9.720 2.267 9.435 88.2121 3.897 1.972 19.079 15.902 27.825																	
Tick Loss																		
Model	Tick Loss																	
	AUT	ISL	GRR	DNK	ESP	NOR	FIN	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC
DFM	GARCH																	
	0.683	0.376	0.941	0.134	0.077	0.145	0.077	0.145	0.077	0.145	0.077	0.145	0.077	0.145	0.077	0.145	0.077	0.145
DFM	CL-GARCH																	
	0.684	0.343	0.986	0.134	0.086	0.136	0.077	0.145	0.077	0.145	0.077	0.145	0.077	0.145	0.077	0.145	0.077	0.145
ARI	GARCH																	
	0.676	0.374	0.127	0.187	0.086	0.125	0.170	0.087	0.073	0.150	0.102	0.107	0.082	0.316	0.087	0.152	0.076	0.191
ARI	CL-GARCH																	
	0.681	0.368	0.169	0.135	0.100	0.185	0.181	0.090	0.087	0.147	0.175	0.084	0.1075	0.296	0.095	0.164	0.070	0.210
QR	0.673 0.465 0.081 0.079 0.082 0.125 0.192 0.097 0.065 0.173 0.173 0.173 0.173 0.173 0.173 0.173 0.173 0.173 0.173																	

Table 13: Country specific hypothesis testing: three and four step ahead forecast horizons, out of sample analysis

		Coverage																		
		(b) 3 step ahead								(b) 4 step ahead										
		Coverage				Coverage				Coverage				Coverage						
Model		AUT	ISL	GBR	DNK	ESP	NOR	FIN	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC	
DFM	GARCH	2.974	2.974	3.861	3.861	2.974	6.405	3.861	5.198	1.287	2.974	3.861	1.287	3.861	11.683	2.974	6.405	2.974	6.405	
	CL-GARCH	2.974	2.974	3.861	3.861	2.974	6.405	3.861	5.198	1.287	2.974	3.861	1.287	3.861	11.683	2.974	6.405	2.974	6.405	
ARI	GARCH	5.198	2.974	5.198	3.861	5.198	6.405	3.861	5.198	6.405	3.861	5.198	6.405	3.861	5.198	6.405	3.861	5.198	6.405	
	CL-GARCH	6.405	3.861	5.198	5.198	5.198	6.405	3.861	5.198	6.405	3.861	5.198	6.405	3.861	5.198	6.405	3.861	5.198	6.405	
QR		7.792	16.831	16.831	12.957	12.957	12.957	3.861	20.779	11.683	18.181	11.683	18.181	11.683	3.861	5.198	10.896	12.957	12.957	
Unconditional Coverage Test																				
Model		AUT	ISL	GBR	DNK	ESP	NOR	FIN	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC	
DFM	GARCH	1.197	1.197	0.213	0.213	0.001	0.213	0.001	0.213	0.001	0.213	0.001	0.213	0.001	0.213	0.001	0.213	0.001	0.213	
	CL-GARCH	0.285	0.285	0.644	0.644	0.644	0.379	0.776	0.285	0.644	0.644	0.644	0.776	0.644	0.206	0.285	0.566	0.285	0.566	
ARI	GARCH	0.001	1.197	0.001	0.213	0.001	0.001	0.001	0.001	0.001	0.213	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	
	CL-GARCH	0.379	0.285	0.379	0.644	0.379	0.289	0.644	0.379	0.644	0.644	0.644	0.644	0.379	0.174	0.076	0.285	0.379	0.644	
QR		0.318	0.318	0.001	0.001	0.001	0.318	0.001	0.001	0.001	0.213	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	
Dynamic Quantile Test																				
Model		AUT	ISL	GBR	DNK	ESP	NOR	FIN	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC	
DFM	GARCH	17.021	2.729	33.429	7.319	17.779	7.846	4.632	13.473	2.071	20.648	0.933	3.195	33.688	24.664	17.123	40.429	17.283	22.281	
	CL-GARCH	0.018	0.943	0.000	0.376	0.030	0.892	0.787	0.014	0.958	0.054	0.959	0.868	0.000	0.009	0.166	0.000	0.039	0.292	
ARI	GARCH	47.297	2.727	47.293	33.238	47.702	5.822	9.934	28.778	18.079	10.319	21.949	4.928	54.278	23.55	17.212	20.480	14.850	26.948	
	CL-GARCH	0.000	0.895	0.000	0.000	0.000	0.357	0.192	0.002	0.016	0.21	0.025	0.212	0.000	0.947	0.161	0.046	0.047	0.003	
QR		59.872	69.164	79.565	38.804	181.709	11.683	7.788	46.423	26.727	8.770	106.979	24.041	104.795	88.295	11.309	16.059	22.688	49.894	
Tick Loss																				
Model		AUT	ISL	GBR	DNK	ESP	NOR	FIN	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC	
DFM	GARCH	0.001	0.283	0.156	0.102	0.080	0.168	0.330	0.855	0.079	0.137	0.166	0.091	0.126	0.260	0.160	0.189	0.076	0.166	
	CL-GARCH	0.097	0.506	0.178	0.1136	0.0830	0.1522	0.1776	0.0863	0.1021	0.1134	0.1655	0.0908	0.143	0.188	0.135	0.180	0.071	0.293	
ARI	GARCH	0.059	0.306	0.172	0.1130	0.0965	0.295	0.1822	0.0950	0.0843	0.185	0.1725	0.1008	0.1004	0.194	0.0895	0.1601	0.077	0.202	
	CL-GARCH	0.0942	0.314	0.1213	0.1096	0.1073	0.1165	0.1388	0.0938	0.0912	0.1457	0.1767	0.0977	0.1209	0.188	0.1000	0.1611	0.079	0.228	
QR		0.0786	0.197	0.0929	0.1045	0.1103	0.1317	0.1555	0.0962	0.0741	0.1333	0.2394	0.0879	0.1321	0.3614	0.0729	0.1658	0.080	0.2923	

Table 15: Country specific hypothesis testing: three and four step ahead forecast horizons, focus on Great Recessions

Coverage		Coverage																				Tick Loss																			
		AR1					AR1					AR1					AR1					AR1					AR1					AR1									
Model	Q	AUT	ISL	GBR	DNK	ESP	NOR	FIN	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC	Model	t-stat	p-value	AUT	ISL	GBR	DNK	ESP	NOR	FIN	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC	
DFM	GARCH	3.333	10.000	6.667	1.667	3.333	3.333	3.333	3.333	3.333	3.333	3.333	1.667	5.000	8.333	1.667	6.667	3.333	8.333	3.333	DFM	t-stat	p-value	AUT	ISL	GBR	DNK	ESP	NOR	FIN <td>USA</td> <td>BEL</td> <td>DEU</td> <td>LUX</td> <td>PRT</td> <td>ITA</td> <td>IRL</td> <td>CHE</td> <td>SWE</td> <td>FRA</td> <td>GRC</td>	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC
	CL-GARCH	1.667	10.000	6.667	1.667	3.333	3.333	3.333	3.333	3.000	1.667	11.667	1.667	6.667	11.667	3.333	3.333	3.333	3.333	3.333	DFM	t-stat	p-value	AUT	ISL	GBR	DNK	ESP	NOR	FIN <td>USA</td> <td>BEL</td> <td>DEU</td> <td>LUX</td> <td>PRT</td> <td>ITA</td> <td>IRL</td> <td>CHE</td> <td>SWE</td> <td>FRA</td> <td>GRC</td>	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC
AR1	GARCH	6.667	5.000	6.667	6.667	5.000	6.667	6.667	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000	AR1	t-stat	p-value	AUT	ISL	GBR	DNK	ESP	NOR	FIN <td>USA</td> <td>BEL</td> <td>DEU</td> <td>LUX</td> <td>PRT</td> <td>ITA</td> <td>IRL</td> <td>CHE</td> <td>SWE</td> <td>FRA</td> <td>GRC</td>	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC	
	CL-GARCH	5.000	6.667	6.667	6.667	5.000	6.667	6.667	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000	AR1	t-stat	p-value	AUT	ISL	GBR	DNK	ESP	NOR	FIN <td>USA</td> <td>BEL</td> <td>DEU</td> <td>LUX</td> <td>PRT</td> <td>ITA</td> <td>IRL</td> <td>CHE</td> <td>SWE</td> <td>FRA</td> <td>GRC</td>	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC	
QR	10.000	13.333	21.667	13.333	23.333	3.333	6.667	18.333	13.333	5.000	21.667	13.333	21.667	13.333	13.333	13.333	13.333	13.333	13.333	QR	t-stat	p-value	AUT	ISL	GBR	DNK	ESP	NOR	FIN <td>USA</td> <td>BEL</td> <td>DEU</td> <td>LUX</td> <td>PRT</td> <td>ITA</td> <td>IRL</td> <td>CHE</td> <td>SWE</td> <td>FRA</td> <td>GRC</td>	USA	BEL	DEU	LUX	PRT	ITA	IRL	CHE	SWE	FRA	GRC	

(a) 3 step ahead

(b) 4 step ahead

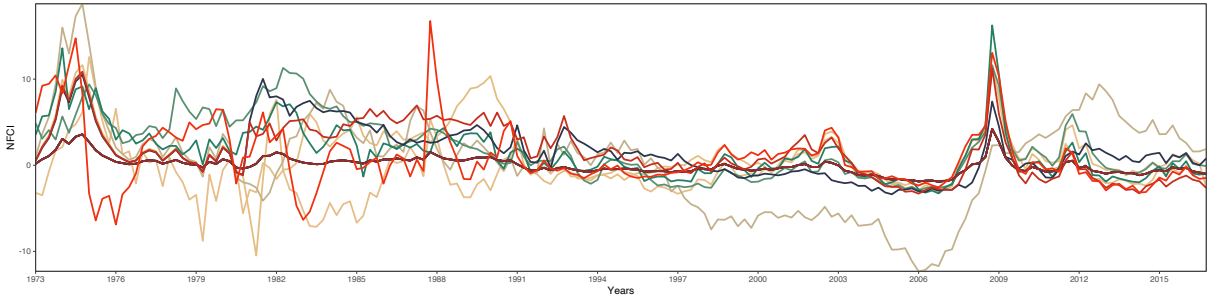
Table 16: Superior predictive ability test, pairwise comparison (in %), out-of-sample analysis and focus on great recession

		Out-Of-Sample Analysis					Great Recession Focus				
h		DFM-G	DFM-CL	AR-G	AR-CL	QR	DFM-G	DFM-CL	AR-G	AR-CL	QR
1	DFM-GARCH	x	22.22	38.89	38.89	44.44	x	22.22	44.44	33.33	33.33
	DFM-CL	77.78	x	44.44	61.11	50.00	66.67	x	55.56	50.00	33.33
	AR-GARCH	61.11	55.56	x	66.67	55.56	33.33	33.33	x	33.33	22.22
	AR-CL	61.11	38.89	33.33	x	50.00	44.44	38.89	50.00	x	44.44
	QR	55.56	50.00	44.44	50.00	x	33.33	27.78	44.44	38.89	x
2	DFM-GARCH	x	61.11	22.22	22.22	33.33	x	33.33	33.33	22.22	33.33
	DFM-CL	22.22	x	22.22	22.22	27.78	55.56	x	44.44	33.33	38.89
	AR-GARCH	27.78	27.78	x	44.44	27.78	16.67	16.67	x	27.78	38.89
	AR-CL	33.33	27.78	44.44	x	38.89	33.33	22.22	61.11	x	38.89
	QR	44.44	44.44	50.00	44.44	x	44.44	44.44	44.44	44.44	x
3	DFM-GARCH	x	61.11	44.44	22.22	38.89	x	44.44	50.00	33.33	44.44
	DFM-CL	33.33	x	27.78	27.78	33.33	44.44	x	61.11	44.44	44.44
	AR-GARCH	22.22	33.33	x	44.44	38.89	11.11	16.67	x	22.22	38.89
	AR-CL	22.22	27.78	50.00	x	44.44	16.67	38.89	77.78	x	44.44
	QR	50.00	50.00	44.44	44.44	x	33.33	38.89	44.44	38.89	x
4	DFM-GARCH	x	33.33	22.22	22.22	33.33	x	38.89	38.89	33.33	44.44
	DFM-CL	44.44	x	11.11	16.67	38.89	44.44	x	38.89	44.44	44.44
	AR-GARCH	61.11	44.44	x	55.56	50.00	22.22	27.78	x	22.22	38.89
	AR-CL	50.00	44.44	44.44	x	50.00	38.89	33.33	55.56	x	44.44
	QR	44.44	44.44	44.44	33.33	x	27.78	22.22	44.44	33.33	x

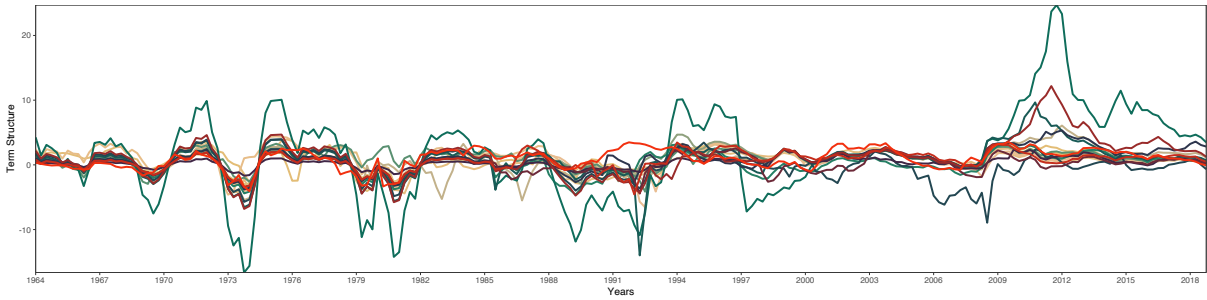
7 Figures

Figure 1: Dynamics of national financial condition and macroeconomics outcomes in Europe, 1970-2018.

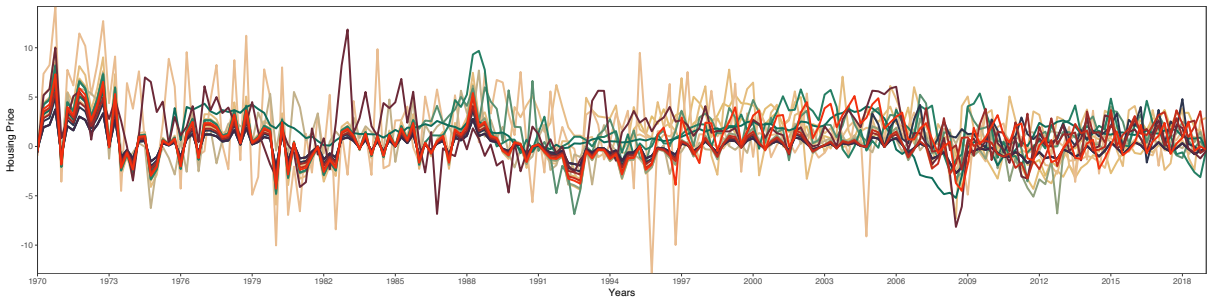
(a) National Financial Condition Index.



(b) Term Structure.



(c) Housing Price.



(d) Gross Domestic Product Growth Rates.

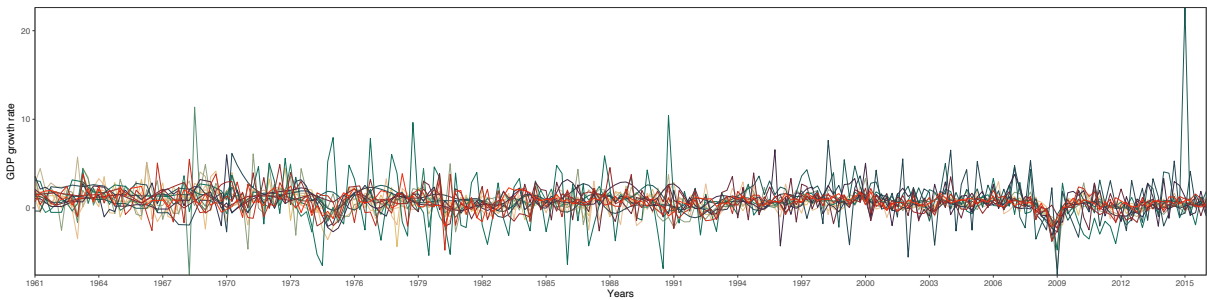


Figure 2: Correlation Heatmap of the dataset for Principal Component Analysis.

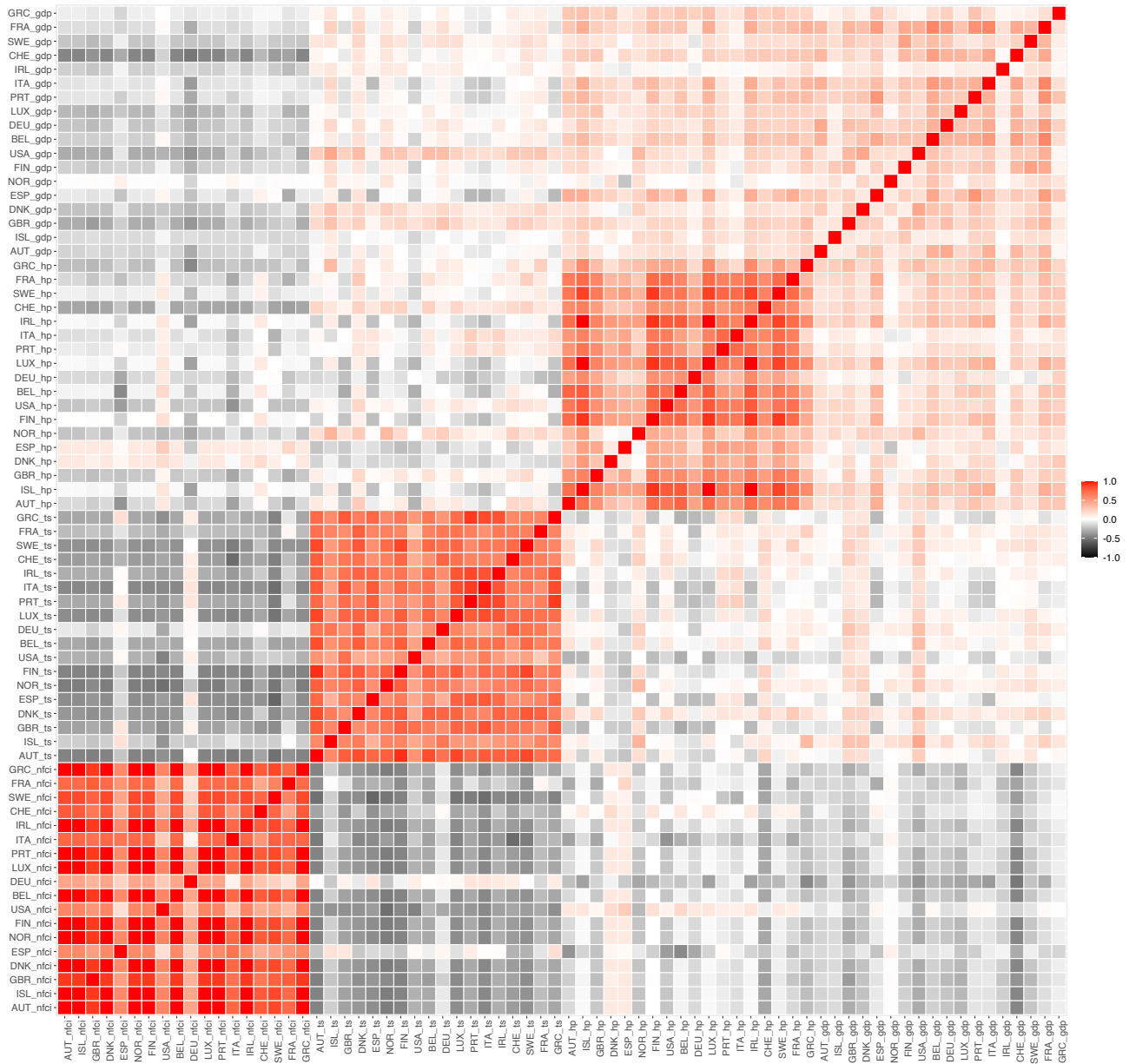
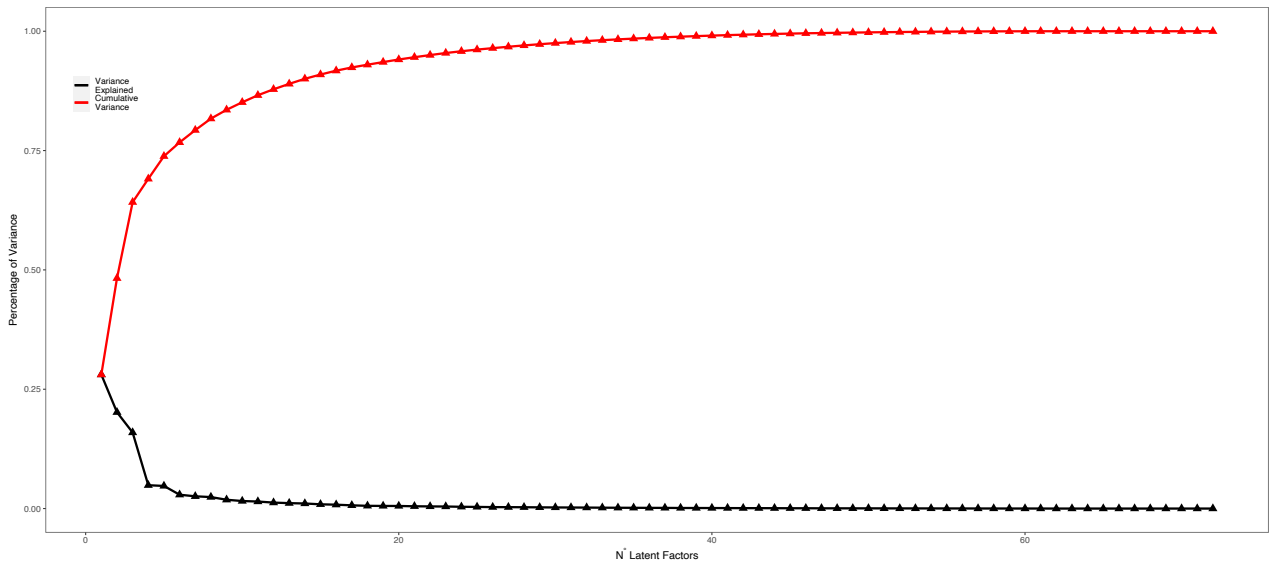


Figure 3: Percentage of variance explained by the latent factors from PCA.

(a) Percentage of variance explained, full sample.



(b) Percentage of variance explained by the first five components, rolling window.

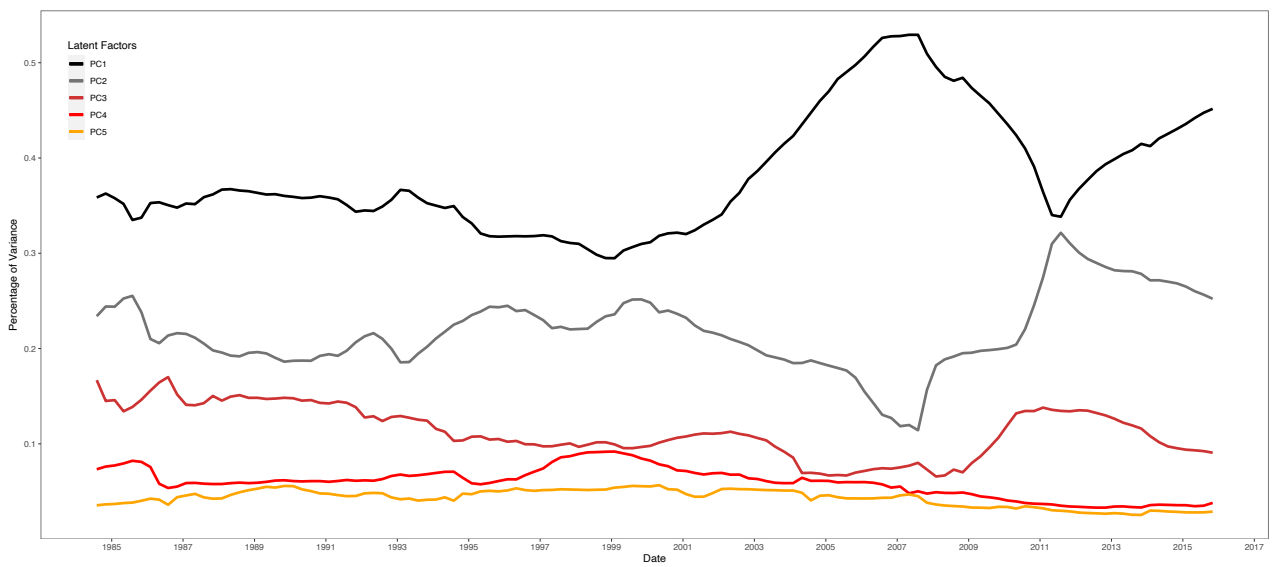
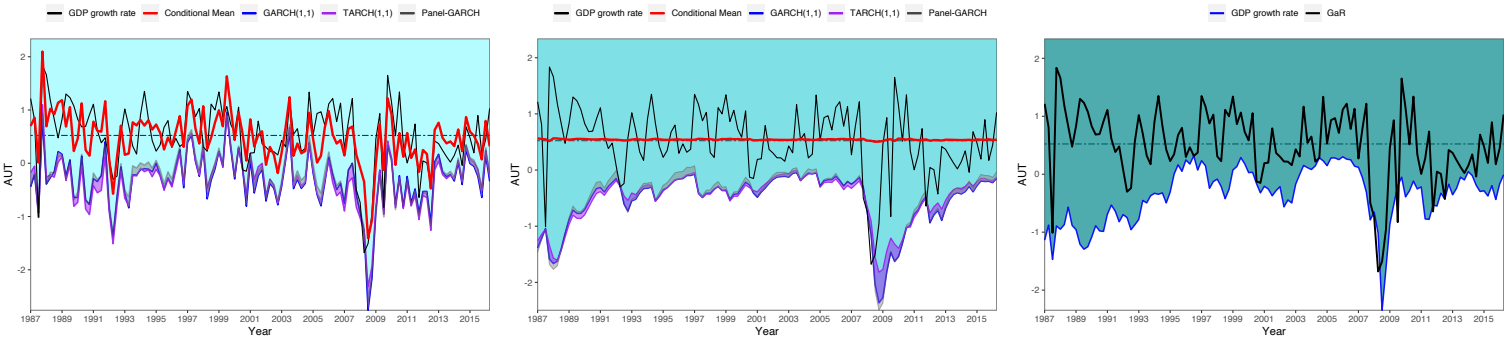
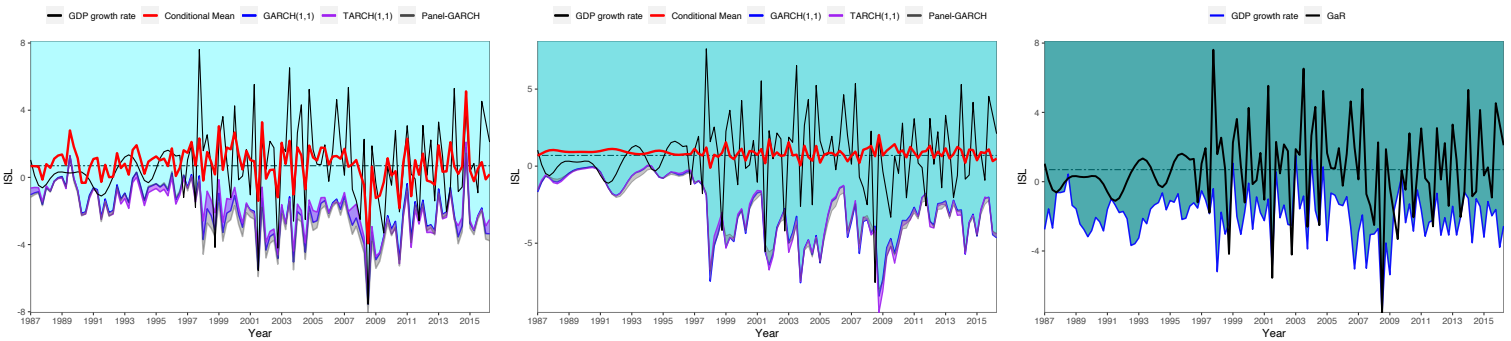


Figure 4: Comparison of GaR predictions (1/5) between Dynamic Factor Model (left), AR(1)-Model (center), and Quantile Regression (right)

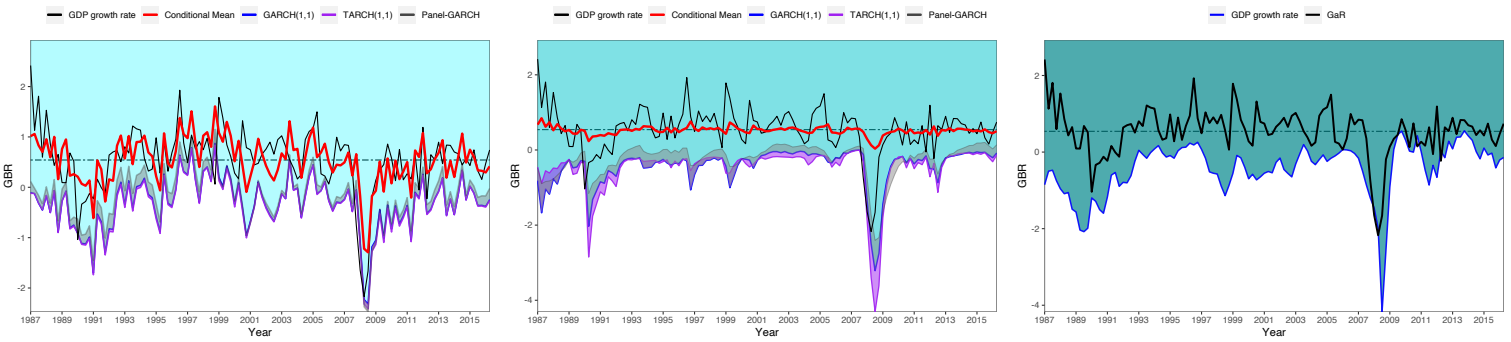
(a) Austria



(b) Iceland



(c) Great Britain



(d) Denmark

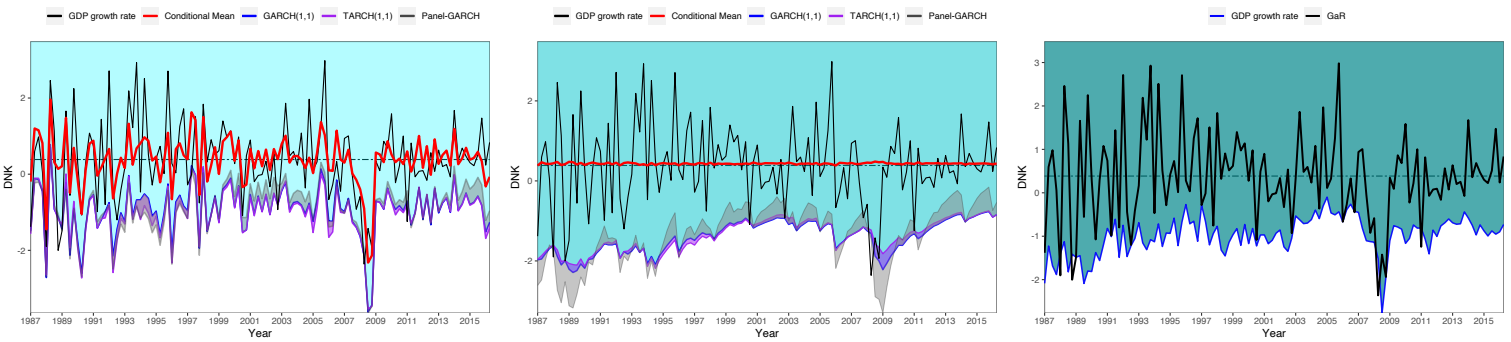
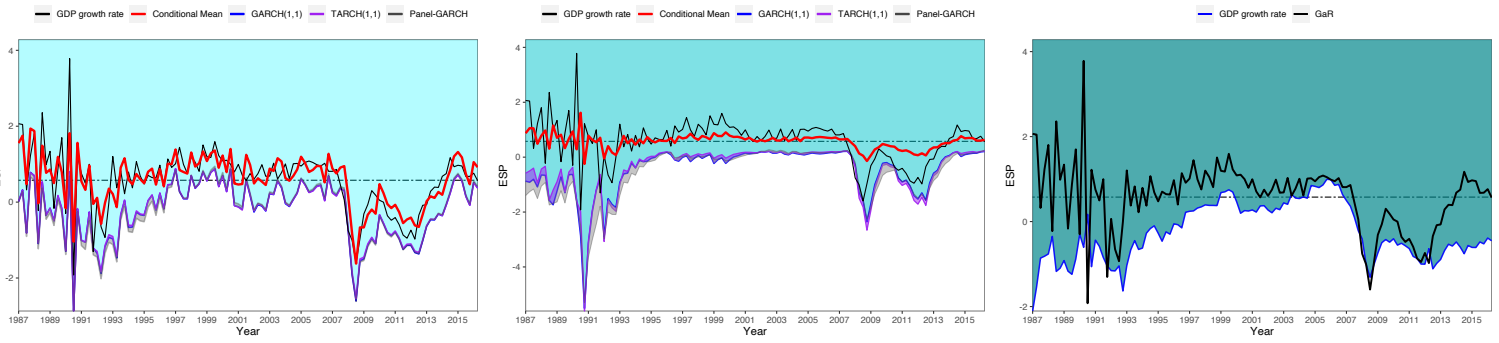
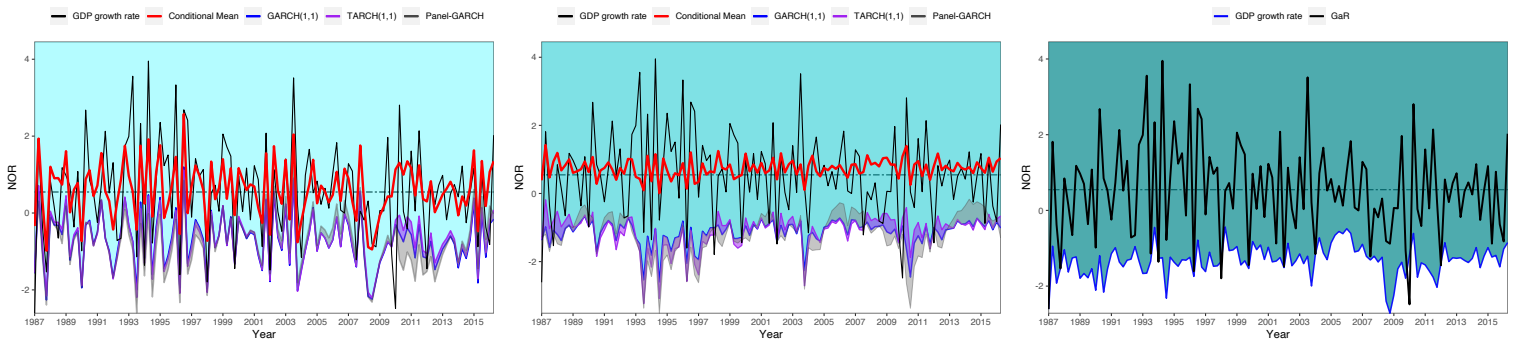


Figure 5: Comparison of GaR predictions (2/5) between Dynamic Factor Model (left), AR(1)-Model (center), and Quantile Regression (right)

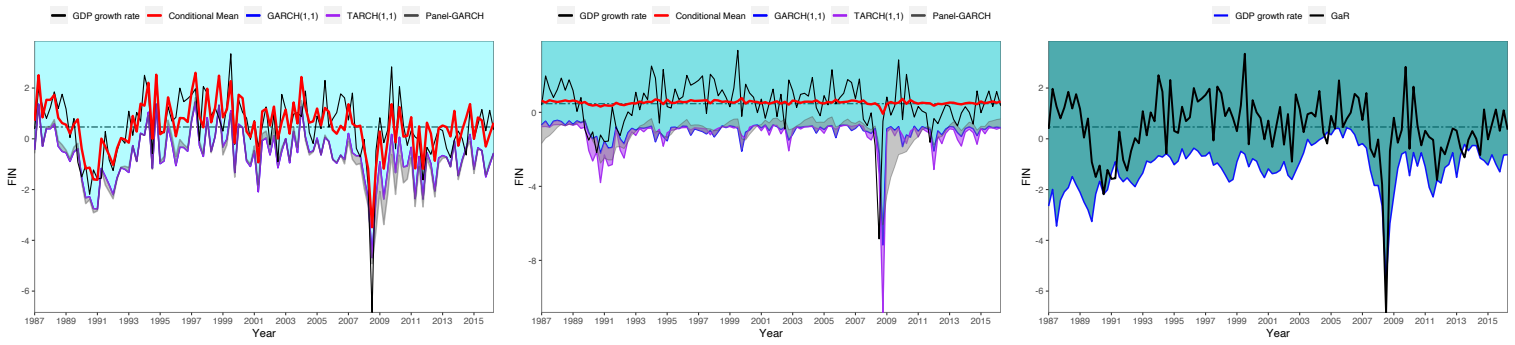
(a) Spain



(b) Norway



(c) Finland



(d) United States

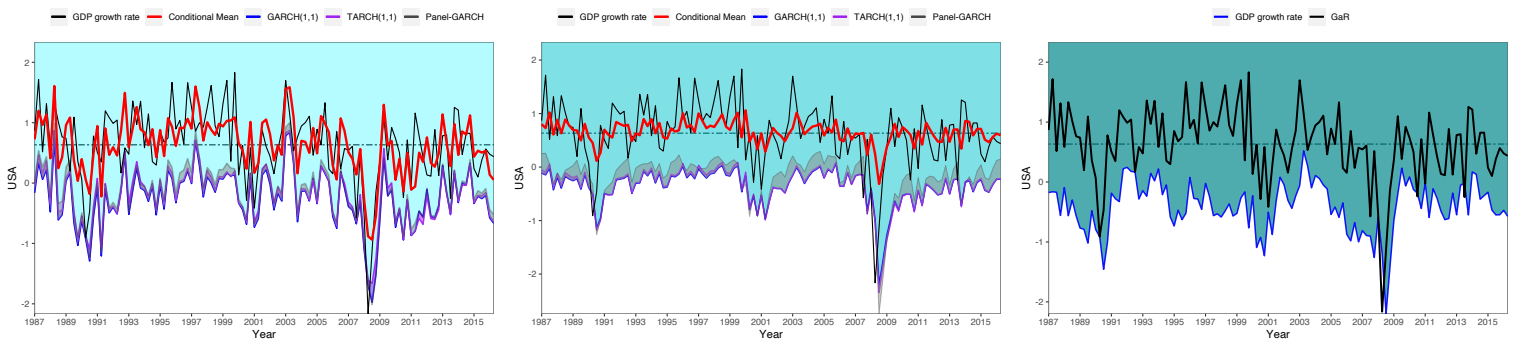
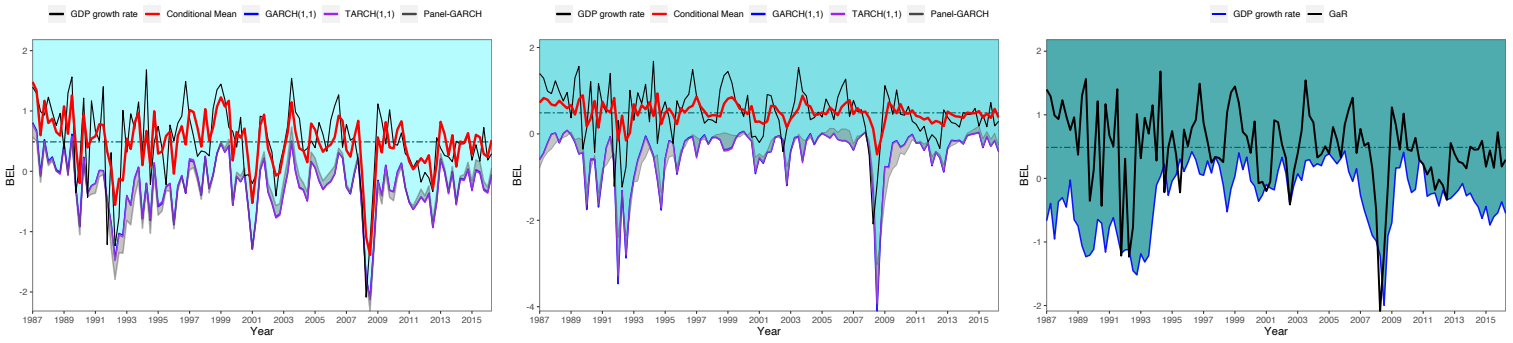
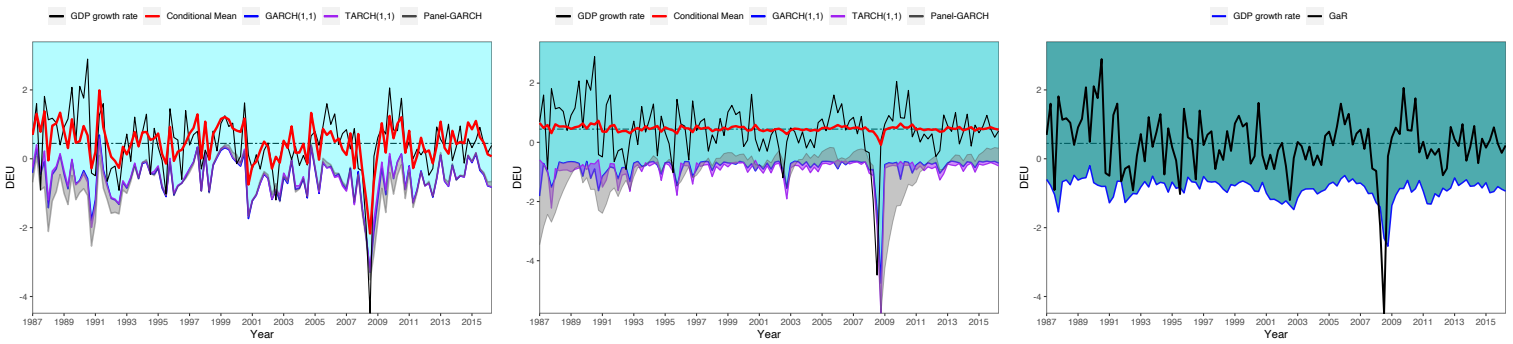


Figure 6: Comparison of GaR predictions (3/5) between Dynamic Factor Model (left), AR(1)-Model (center), and Quantile Regression (right)

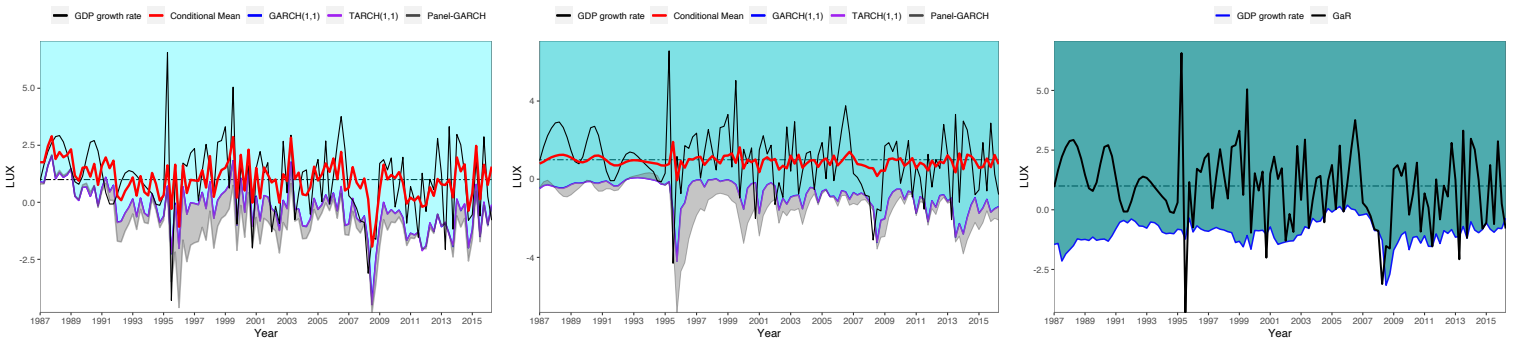
(a) Belgium



(b) Germany



(c) Luxembourg



(d) Portugal

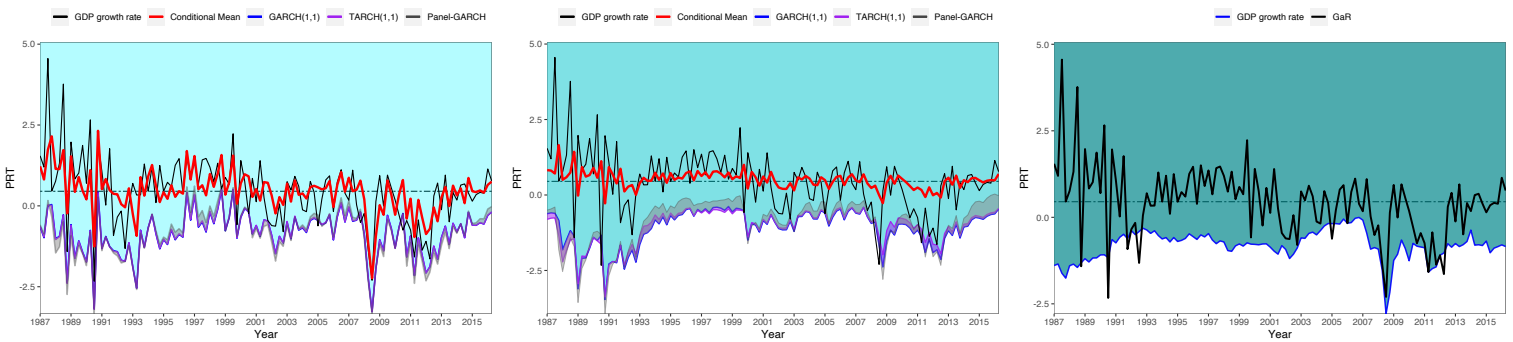
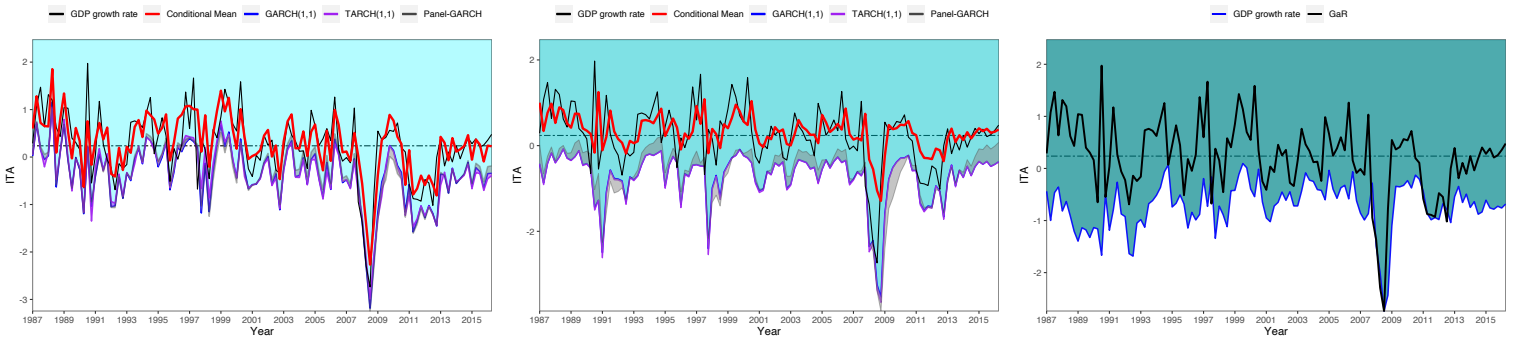
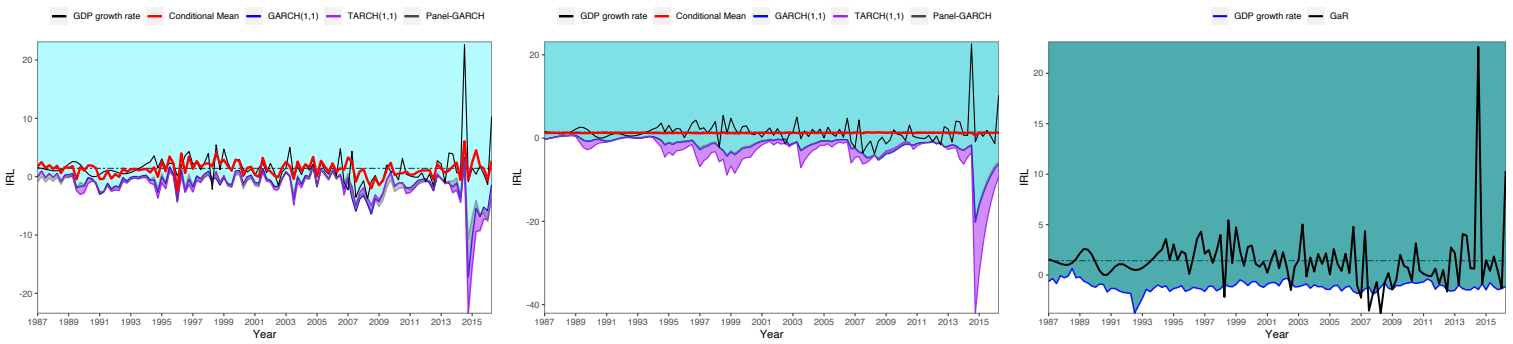


Figure 7: Comparison of GaR predictions (4/5) between Dynamic Factor Model (left), AR(1)-Model (center), and Quantile Regression (right)

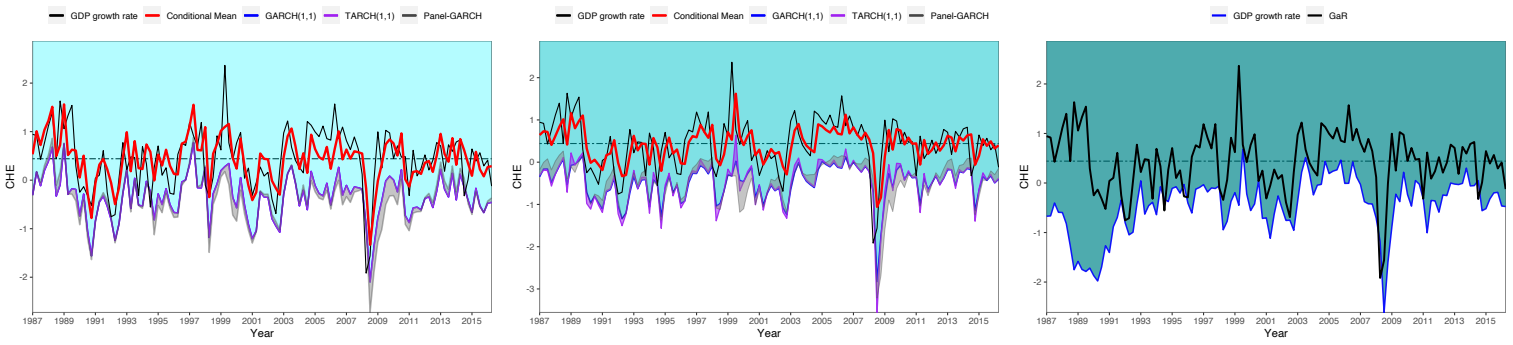
(a) Italy



(b) Ireland



(c) Switzerland



(d) Sweden

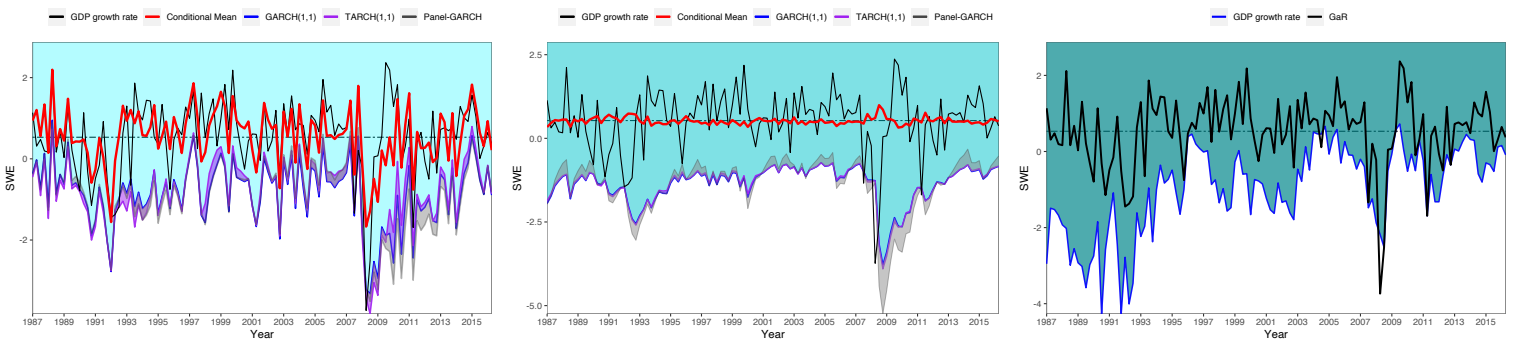
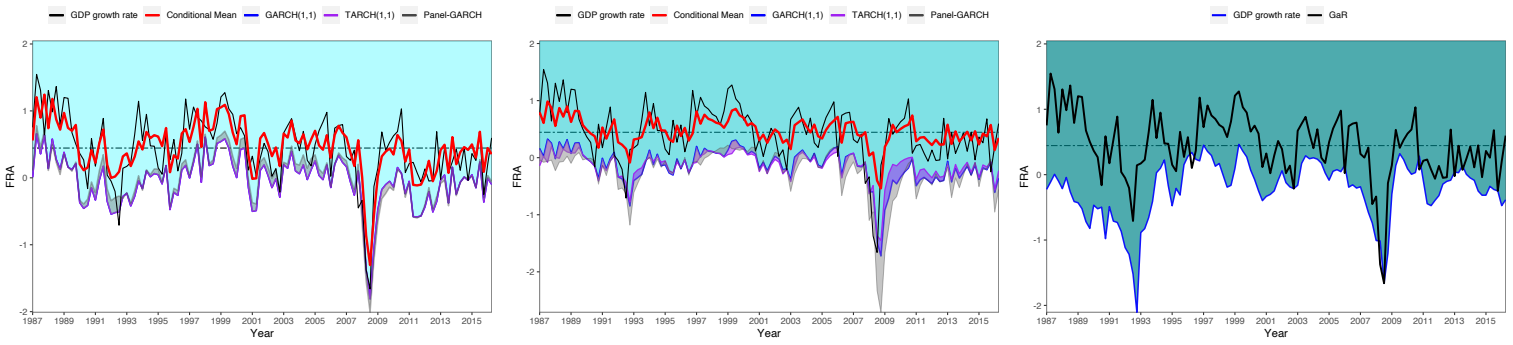
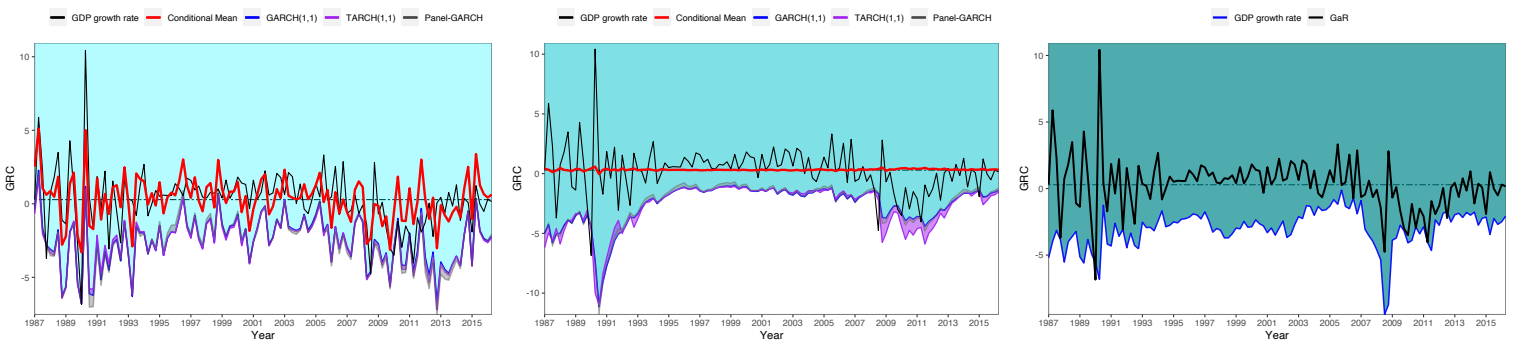


Figure 8: Comparison of GaR predictions (5/5) between Dynamic Factor Model (left), AR(1)-Model (center), and Quantile Regression (right)

(a) France



(b) Greece



Appendix

A Reduced Form VAR Estimation

Now we are going to better explain the procedures that we implemented for the estimation by OLS of the reduced form-VAR. First, the starting point of the analysis is the reduced form-VAR:

$$Y_t = A_1 Y_{t-1} + \dots + A_p Y_{t-p} + u_t. \quad (41)$$

that we rewrite in its companion form representation:

$$\xi_t = \mathbb{A} \xi_{t-1} + U_t \quad (42)$$

where

$$\xi_t = \begin{bmatrix} Y_t \\ Y_{t-1} \\ Y_{t-2} \\ \vdots \\ Y_{t-p} \end{bmatrix} \quad \mathbb{A} = \begin{bmatrix} A_1 & A_2 & \dots & A_{p-1} & A_p \\ I_k & 0_k & \dots & 0 & 0 \\ 0 & I_k & \dots & 0 & 0 \\ \vdots & & & \vdots & \\ 0 & 0 & \dots & I_k & 0 \end{bmatrix} \quad \xi_{t-1} = \begin{bmatrix} Y_{t-1} \\ Y_{t-2} \\ Y_{t-3} \\ \vdots \\ Y_{t-p-1} \end{bmatrix} \quad U_t = \begin{bmatrix} u_t \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \quad (43)$$

The vectors ξ_t , ξ_{t-1} and U_t have dimension $(kp \times 1)$ with k that is the number of endogenous variables of the VAR and p the number of lags, while the companion matrix \mathbb{A} has dimension $(kp \times kp)$. The estimation of \mathbb{A} is straightforward:

$$\hat{\mathbb{A}} = (\xi_t \xi_{t-1}') (\xi_{t-1} \xi_{t-1}')^{-1}. \quad (44)$$

Since our stochastic process $\{\xi_t\}_{t \geq 0}$ is weakly stationary by construction, we express the companion form VAR in equation (42) as an infinite moving average:

$$\xi_t = \sum_{h=0}^{\infty} \mathbb{A}^h U_{t-h} \quad (45)$$

Lastly, we can recover the infinite-order representation of our VAR of order p by pre-multiplying both sides of equation (45) by $J = [I_k \quad 0_{k \times k(p-1)}]$:

$$J \xi_t = \sum_{h=0}^{\infty} J \hat{\mathbb{A}}^h J' J U_{t-h} \quad \text{with: } J' J = I_{kp} \quad (46)$$

$$Y_t = \sum_{h=0}^{\infty} A_h u_{t-h} \quad (47)$$

Being that the matrix \mathbb{A} has been obtained via OLS, then we are able to isolate A_h (composed by the first $(k \times kp)$ elements of the companion matrix \mathbb{A}) using the indicator

matrix J . Regarding the dynamic factor model illustrated in the theoretical section, we were able to consistently estimate $\delta(L)$ and $\Psi(L)$.

B Factor-GARCH specification for the AR(1)

In our in-sample analysis, after modelling an AR(1) for the conditional mean of the process as specified by Brownlees and Souza (2021), we also implemented the Factor-GARCH structure of Engle, Ng, and Rothschild (1990) for the conditional variance $\sigma_{it+1|t}^2$. There are two reasons about why we decided to explain this specification in a separate appendix: first, to avoid confusion with our Dynamic Factor-GARCH Model, and second, because we did not use these specification for the out of sample analysis of the model. That being said, we are going to provide an overview about its implementation and we will illustrate the figures of the in-sample GaR estimation.

Given the residual $\kappa_{it} = Y_{it} - \mu_{it|t-1}$, the Factor GARCH structure is the following:

$$\begin{aligned}
 r_{jt} &= \sqrt{\sigma_{jt|t-1}^2} Z_{jt} \quad \text{with: } j = 1, \dots, r \\
 \sigma_{jt|t-1}^2 &= \omega_j + \alpha_j r_{jt-1}^2 + \beta_j \sigma_{jt-1|t-2}^2 \\
 \kappa_{it} &= \lambda_{1i} r_{1t} + \dots + \lambda_{ri} r_{rt} + \sqrt{\sigma_{it|t-1}^2} Z_{it} \quad \text{with: } i = 1, \dots, N \\
 \sigma_{it|t-1}^2 &= \omega_j + \alpha_j (\kappa_{it} - \lambda_{1i} r_{1t} - \dots - \lambda_{ri} r_{rt})^2 + \beta_j \sigma_{it-1|t-2}^2
 \end{aligned} \tag{48}$$

where r represents the number of factors¹⁶, λ_{ji} is the factor loadings of the j^{th} factor on the i^{th} GDP growth rates. Remarkably, the residual κ_{it} is affected by the contemporaneous realizations of the factor, and not by their previous realization like in the Dynamic Factor-GARCH model. The estimation procedure of the models is divided in three steps:

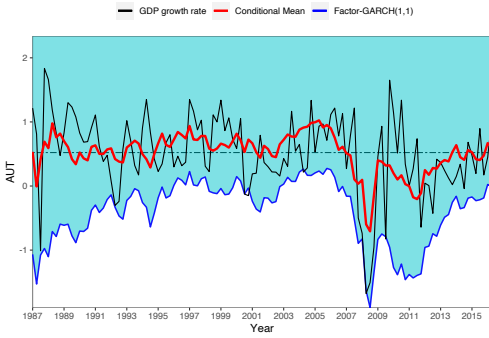
1. Estimation of the GARCH model of the factors through Maximum Likelihood;
2. Estimation of the factor loading by OLS, regressing κ_{it} on the latent factors $\{r_{1t}, \dots, r_{rt}\}$;
3. Estimation of the conditional variance $\sigma_{it|t-1}^2$ by Maximum Likelihood using the residual $\eta_{it} = \kappa_{it} - \lambda_{1i} r_{1t} - \dots - \lambda_{ri} r_{rt}$.

In figures 9 - 10 we display the GaR forecasts produced by using this AR(1) model specification. While its performance has been already analyzed in the in-sample analysis, it is important to notice how this specification produces better fitted values for the conditional mean of the GDP growth rates vis-à-vis all the other AR(1) specifications; moreover, it produces less dispersed GaR forecasts, which follow the macroeconomic fluctuations more closely.

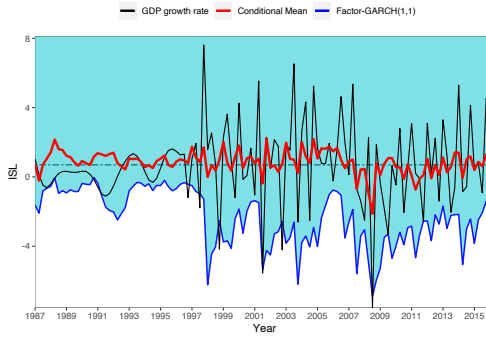
¹⁶We used the same 5 factors obtained from the PCA and used in the Dynamic Factor-GARCH Model.

Figure 9: Comparison of GaR predictions (1/2) of the Factor-GARCH specification for the AR(1).

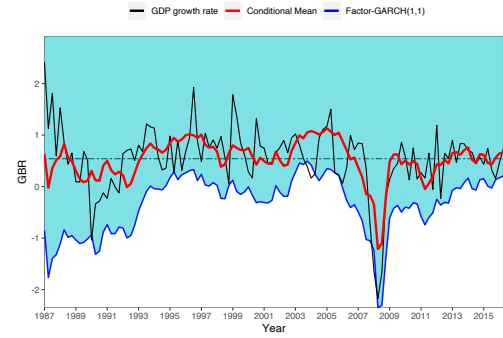
(a) Austria



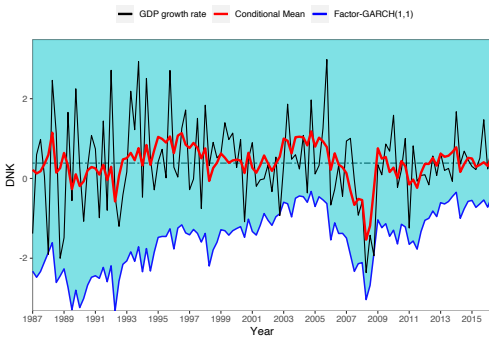
(b) Iceland



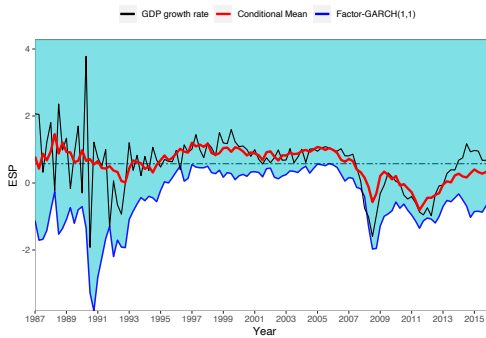
(c) Great Britain



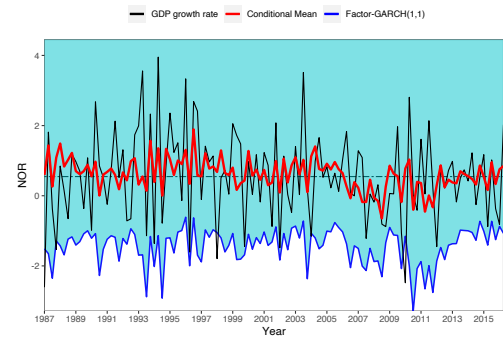
(d) Denmark



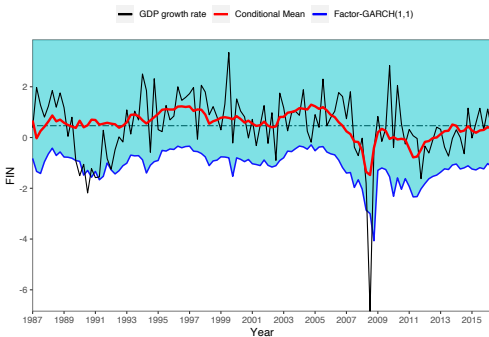
(e) Spain



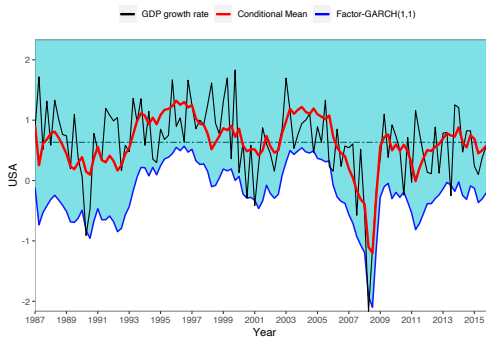
(f) Norway



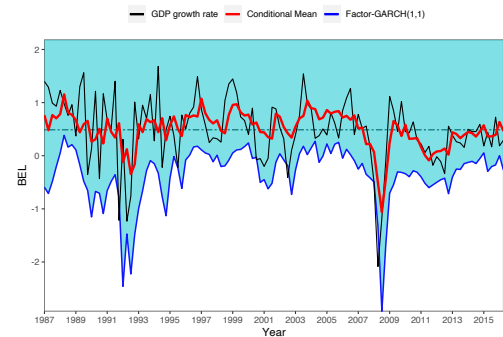
(g) Finland



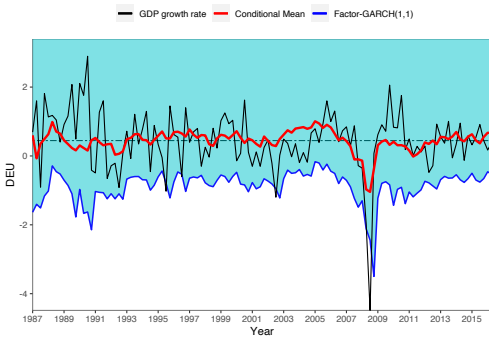
(h) United States



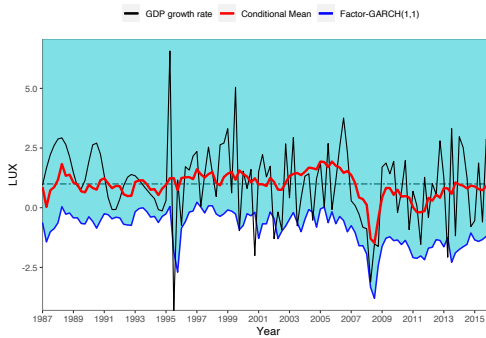
(i) Belgium



(j) Germany



(k) Luxembourg



(l) Portugal

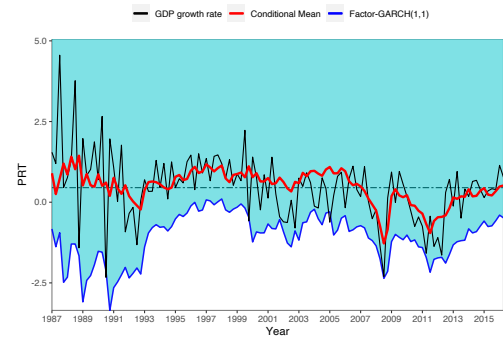
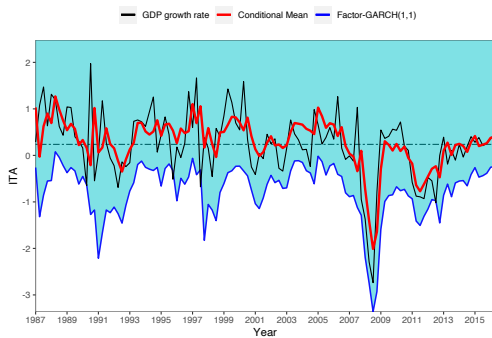
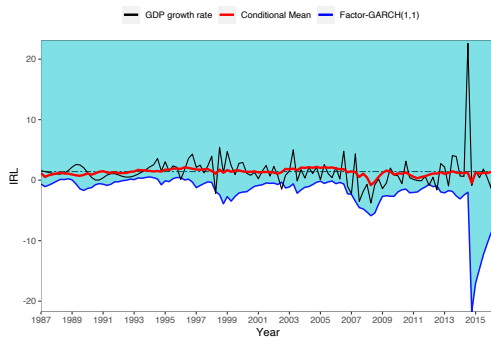


Figure 10: Comparison of GaR predictions (2/2) of the Factor-GARCH specification for the AR(1).

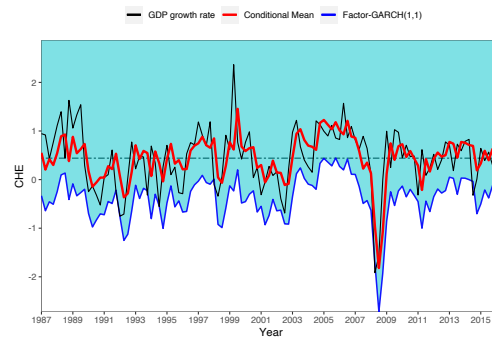
(a) Italy



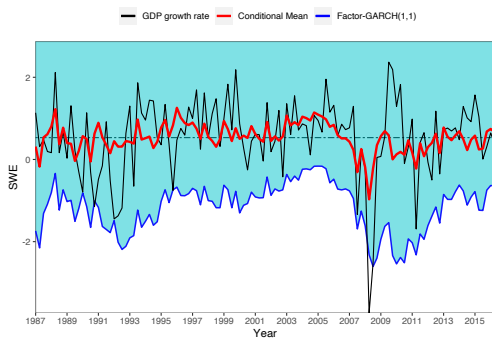
(b) Ireland



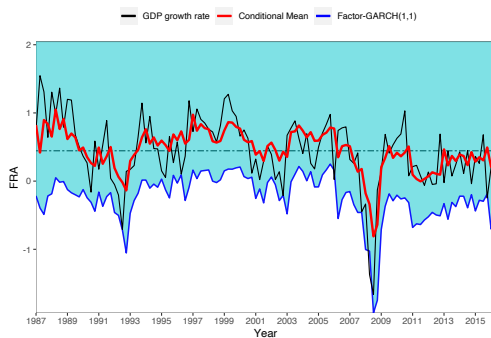
(c) Switzerland



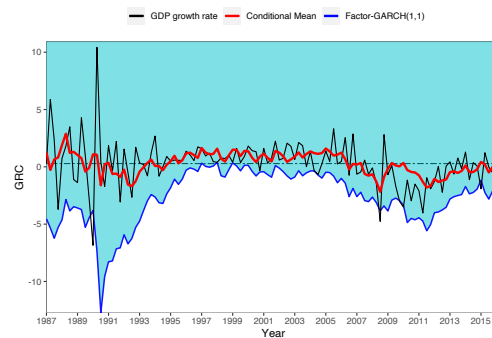
(d) Sweden



(e) France



(f) Greece



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